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Modeling Regional Interdependencies Using a Global Error-Correcting Macroeconometric Model

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Financial institutions are ultimately exposed to macroeconomic fluctuations in the global economy. This article proposes and builds a compact global model capable of generating forecasts for a core set of macroeconomic factors (or variables) across a number of countries. The model explicitly allows for the interdependencies that exist between national and international factors. Individual region-specific vector error-correcting models are estimated in which the domestic variables are related to corresponding foreign variables constructed exclusively to match the international trade pattern of the country under consideration. The individual country models are then linked in a consistent and cohesive manner to generate forecasts for all of the variables in the world economy simultaneously. The global model is estimated for 25 countries grouped into 11 regions using quarterly data over 1979Q1–1999Q1. The degree of regional interdependencies is investigated via generalized impulse responses where the effects of shocks to a given variable in a given country on the rest of the world are provided. The model is then used to investigate the effects of various global risk scenarios on a bank's loan portfolio.

KEY WORDS: Credit loss distribution; Global interdependencies; Global macroeconometric modeling; Global vector error-correcting model; Risk management.

1. INTRODUCTION

Increased globalization of the world economy has important consequences for the conduct of monetary and financial policies by central bankers and risk management by commercial bankers. In setting interest rates, more than ever before, central bankers need to allow for the interrelationships that exist between their economy and the rest of the world. In a commercial banking context, the risk analysis of a bank's financial activities needs to take into account domestic economic conditions as well as the economic conditions of countries that directly or indirectly influence the loss distribution of banks' loan portfolios. Thus both constituencies would benefit from working with a global macroeconometric model that is capable of generating forecasts for a core set of macroeconomic factors for a set of regions and countries to which they have risk exposures and that explicitly allows for interconnections and interdependencies that exist between national and international factors in a coherent and consistent manner.

This article aims to provide such a global modeling framework by making use of recent advances in the analysis of cointegrating systems. So far, applications of the cointegrating approach have been confined to a single country covering only some of the key macroeconomic variables (see, e.g., King, Plosser, Stock, and Watson 1991; Mellander, Vredin, and Warne 1992; Crowder, Hoffman, and Rasche 1999; and Garratt, Lee, Pesaran, and Shin 2000, 2003a). Although in principle it is possible to extend the approach to modeling interrelationships across different economies, in practice such a strategy will not be feasible due to data limitations. In an unrestricted vector autoregressive (VAR) model covering N regions the number of unknown parameters rises with N, and even if we focus on a few key macroeconomic indicators such as output, inflation, interest rate, and exchange rate, then there will be $p(kN-1)$ unknown parameters (not counting intercepts or other deterministic/exogeneous variables) to be estimated for each equation, where p is the order of the VAR and k is the number of the endogenous variables per region. For example, in the case of a world economy composed of 10 regions with $p=2$, and $k=5$, there will be at least as many as 98 unknown coefficients to be estimated per equation, with the available quarterly time series being of the same order of magnitude for advanced economies and often much less in the case of other regions.

In view of these difficulties, global forecasting models are often formed by linking up of the traditional, typically large-scale, macroeconometric models originally developed for the national economies. A prominent example of this approach is Lawrence Klein's Project Link, adopted by the United Nations. A similar approach, albeit on a smaller scale, has been followed by international agencies, such as the International Monetary Fund

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(IMF) and Organisation for Economic Cooperation and Development (OECD). The National Institute's Global Econometric Model (NiGEM) estimates/calibrates a common model structure across OECD countries, China, and a number of regional blocks. The country/region-specific models in NiGEM are still quite large, each comprising 60–90 equations with 30 key behavioral relations (Barrell, Dury, Hurst, and Painl 2001). Global models with limited geographical coverage have also been developed. For example, Rae and Turner (2001) developed a small forecasting model covering the United States, the European area, and Japan. These contributions provide significant insights into the important interlinkages that exist among major world economies and have proven essential in global forecasting. Nevertheless, they are difficult to use for risk management purposes and do not adequately address the important financial interlinkages that exist among the world's major economies.

In this article we propose a new approach to modeling the global economy that avoids some of these limitations while at the same time providing a consistent and flexible framework for use in a variety of applications, such as risk management. We first estimate individual country- (or region-)specific vector error-correcting models (VECMs), where such domestic macroeconomic variables as gross domestic product (GDP), the general price level, the level of short-term interest rate, exchange rate, equity prices (when applicable), and money supply are related to corresponding foreign variables constructed to match the international trade pattern of the country under consideration. For purposes of estimation and inference, these country-specific foreign variables can be treated as weakly exogenous (or long-run forcing) for most economies when N is sufficiently large and the idiosyncratic shocks are weakly correlated; a notable exception of course being the U.S. economy. The model for the U.S. can be estimated by treating most of the variables as endogenous. The individual country models are then combined in a consistent and cohesive manner to generate forecasts or impulse response functions for *all* of the variables in the world economy simultaneously.

We use the estimated global model as the economic engine for generating conditional loss distributions of a credit portfolio. Business cycle fluctuations can have a major impact on credit portfolio loss distributions. Carey (2002), using resampling techniques, showed that mean losses of a typical portfolio during a recession such as 1990/91 in the U.S. are about the same as losses in the .5% tail during an expansion. Bangia, Diebold, Kronimus, Schagen, and Schuermann (2002), using a regime-switching approach, found that capital held by banks over a 1-year horizon needs to be 25–30% higher in a recession than in an expansion. The basic idea of our approach is to make more explicit the linkage between a bank's credit exposures and the underlying international macroeconomic conditions.

The article is organized as follows. Section 2 sets out the country/region-specific models and establishes the interlinkages between each of the economies and the rest of the world through trade-based weighting matrices. Section 3 then combines the different country-specific VECM models, and provides a complete solution of the global VAR (GVAR). Section 4 examines the error-correcting properties of the global model and shows that the number of long-run relationships in the global model cannot exceed the sum of the long-run relations of the region specific models. Section 5 discusses dynamic and stability properties of the GVAR model. Section 6 derives impulse response functions for the analysis of shocks in one country on the macroeconomic variables in other countries. Section 7 considers the estimation problem of the country-specific models, and Section 8 discusses the practical issues surrounding the construction of regional aggregates. To ensure maximum global coverage while keeping the risk analysis manageable, it is often necessary to work at regional levels, and Section 8 also addresses the aggregation bias that this may entail and explores ways to minimize such a bias. Section 9 sets out an empirical illustration of the approach, and estimates and analyzes a GVAR model in seven countries (U.S., U.K., Germany, France, Italy, China, and Japan) and four regions (Western Europe, Middle East, Southeast Asia, and Latin America). This section also reports a number of impulse response functions demonstrating how the model could be used in the analysis of the transmission of stock market and interest rate shocks from one region to the rest of the world economy. Section 10 links a firm's return (and default) process to macroeconomic (systematic) variables and then proceeds to generate loss distributions *conditional* on the estimated GVAR specification from Section 9, as well as analyze the impact of economic shocks on loss. Section 11 offers some concluding remarks. The Appendix provides a summary of data sources used, as well as a brief account of how the regional series were constructed.

2. COUNTRY–SPECIFIC MODELS

We assume that there are $N+1$ countries (or regions) in the global economy, indexed by $i = 0, 1, 2, \ldots, N$. We adopt country 0 as the reference country (the U.S. seems an obvious choice). For each country/region, we assume that the country-specific variables are related to the global economy variables measured as country-specific weighted averages of foreign variables plus deterministic variables, such as time trends, and global (weakly) exogenous variables, such as oil prices. For simplicity, here we confine our exposition to a first-order dynamic specification that relates the $k_i \times 1$ country-specific factors/variables, $\mathbf{x}_{it}$, to $\mathbf{x}_{it}^*$, a $k_i^* \times 1$ vector of foreign variables specific to country i (defined later) and write

$$\mathbf{x}_{it} = \mathbf{a}_{i0} + \mathbf{a}_{i1}t + \boldsymbol{\Phi}_i\mathbf{x}_{i,t-1} + \boldsymbol{\Lambda}_{i0}\mathbf{x}_{it}^* + \boldsymbol{\Lambda}_{i1}\mathbf{x}_{i,t-1}^* + \boldsymbol{\varepsilon}_{it},$$
$$t = 1, 2, \ldots, T, i = 0, 1, 2, \ldots, N, \quad (1)$$

where $\boldsymbol{\Phi}_i$ is a $k_i \times k_i$ matrix of lagged coefficients, $\boldsymbol{\Lambda}_{i0}$ and $\boldsymbol{\Lambda}_{i1}$ are $k_i \times k_i^*$ matrices of coefficients associated with the foreign-specific variables, and $\boldsymbol{\varepsilon}_{it}$ is a $k_i \times 1$ vector of idiosyncratic country-specific shocks. In the special case where $\boldsymbol{\Lambda}_{i0} = \boldsymbol{\Lambda}_{i1} = \mathbf{0}$, this model reduces to a standard VAR process of order 1, VAR(1). However, in the presence of foreign-specific variables, (1) is an augmented VAR model, which we denote by VARX*(1, 1).

We assume that the idiosyncratic shocks, $\boldsymbol{\varepsilon}_{it}$, are serially uncorrelated with mean $\mathbf{0}$ and a nonsingular covariance matrix, $\boldsymbol{\Sigma}_{ii} = (\sigma_{ii,\ell s})$, where $\sigma_{ii,\ell s} = \mathrm{cov}(\varepsilon_{i\ell t}, \varepsilon_{ist})$, or, written more compactly,

$$\boldsymbol{\varepsilon}_{it} \backsim \mathrm{iid}(\mathbf{0}, \boldsymbol{\Sigma}_{ii}). \quad (2)$$

The assumption that the country-specific variance-covariance matrices, $\boldsymbol{\Sigma}_{ii}$, $i = 0, 1, 2, \dots, N$, are time invariant can be relaxed, but for the analysis of quarterly observations, this time invariant assumption may not be overly restrictive. However, when the focus of the analysis is on contagion or spillover effects resulting from systemic risk, it may be necessary to consider regime-switching models where the parameters of the regional models switch between a "normal" and a "crisis" set of values (for a review, see De Bandt and Hartmann 2000). To accommodate such effects, it would be necessary to specify and estimate nonlinear switching regional models from which a nonlinear global model can be derived, which is beyond the scope of this article (but see Pesaran and Pick 2003 for a discussion of the econometric issues involved in the analysis of single-equation contagion models).

We also allow the idiosyncratic shocks $\boldsymbol{\varepsilon}_{it}$ to be correlated across regions to a limited degree. The exact nature of this dependence is clarified later once the linkages between the country-specific foreign variables, $\mathbf{x}^*_{it}$, and the variables in the rest of the world economic system, namely $(\mathbf{x}_{0t}, \mathbf{x}_{1t}, \dots, \mathbf{x}_{i-1,t}, \mathbf{x}_{i+1,t}, \dots, \mathbf{x}_{Nt})$, are specified.

Typically $\mathbf{x}_{it}$ will include real output (y_{it}), a general price index (p_{it}) or its rate of change, a real equity price index (q_{it}), the exchange rate (e_{it}, measured in terms of a reference currency, say U.S. dollars), an interest rate (ρ_{it}), and real money balances (m_{it}). It may also be necessary to consider other transformations of these underlying variables. For example, as can be seen from our empirical analysis in Section 9, we argue in favor of using the rate of inflation $(p_{it} - p_{i,t-1})$ instead of the price level (p_{it}) and the "real exchange rate" $(e_{it} - p_{it})$ instead of the nominal exchange rate (e_{it}). But to focus ideas here, we set $\mathbf{x}_{it} = (y_{it}, p_{it}, q_{it}, e_{it}, \rho_{it}, m_{it})'$, with $k_i = 6$. We assume that these variables are observed at quarterly frequencies; $y_{it}, p_{it}, q_{it}, e_{it}$, and m_{it} are measured in natural logarithms, and ρ_{it} is an interest rate variable. Output could be measured by real GDP; the general price level, by the consumer price index (CPI); the real equity price index (when available), by broad market indices, such as the S&P500 index in the U.S. or the FTSE100 index in the U.K., deflated by the CPI; the real money supply, by M_0 or M_2 measures of money supply deflated by the CPI; and, finally, the interest rate variable could be either the nominal interest rate on 3-month Treasury Bill rate (or its equivalent) or the (ex post) real interest rate, defined as the nominal rate minus the rate of inflation. For example, a typical set of endogenous variables for country i $(i \neq 0)$, could be

$$
\begin{aligned}
y_{it} &= \ln(GDP_{it}/CPI_{it}), \qquad p_{it} = \ln(CPI_{it}),\\
q_{it} &= \ln(EQ_{it}/CPI_{it}), \qquad m_{it} = \ln(M_{it}/CPI_{it}), \\
e_{it} &= \ln(E_{it}), \qquad \rho_{it} = .25 * \ln(1 + R_{it}/100),
\end{aligned} \tag{3}
$$

where

GDP_{it} = nominal gross domestic product of country i during period t, in domestic currency,
CPI_{it} = consumer price index in country i at time t, equal to 1.0 in a base year (say 1995),
M_{it} = nominal money supply in domestic currency,
EQ_{it} = nominal equity price index,
E_{it} = exchange rate of country i at time t in terms of U.S. dollars,

and

R_{it} = nominal rate of interest per annum, in percent.

Note that in the case of the base economy, $e_{0t} = 0$ and $\mathbf{x}_{0t} = (y_{0t}, p_{0t}, q_{0t}, \rho_{0t}, m_{0t})'$, with $k_0 = 5$. Also, in the case of some of the emerging market economies and the newly constituted economies of the Eastern Europe, where the interest rate and/or the equity price index may not be available over the whole sample period, $\mathbf{x}_{it}$ may be confined to the $y_{it}, p_{it}, e_{it}, m_{it}$, with $k_i = 4$. The foreign variables (indices), denoted by $\mathbf{x}^*_{it}$, is a $k^*_i \times 1$ vector, where $k^*_i = 5$ or 6 in our application, and are constructed as weighted averages, with country/region-specific weights

$$
\begin{aligned}
&\mathbf{x}^*_{it} = (y^*_{it}, p^*_{it}, q^*_{it}, e^*_{it}, \rho^*_{it}, m^*_{it})',\\
&y^*_{it} = \sum_{j=0}^{N} w^y_{ij} y_{jt}, \qquad p^*_{it} = \sum_{j=0}^{N} w^p_{ij} p_{jt},\\
&q^*_{it} = \sum_{j=0}^{N} w^q_{ij} q_{jt}, \qquad e^*_{it} = \sum_{j=1}^{N} w^e_{ij} e_{jt},\\
&\rho^*_{it} = \sum_{j=0}^{N} w^\rho_{ij} \rho_{jt}, \qquad m^*_{it} = \sum_{j=0}^{N} w^m_{ij} m_{jt}.
\end{aligned} \tag{4}
$$

The weights w^y_{ij}, w^p_{ij}, w^q_{ij}, w^e_{ij}, w^ρ_{ij}, and w^m_{ij} for $i, j = 0, 1, \dots, N$, could be based on trade shares (namely, the share of country j in the total trade of country i measured in U.S. dollars) in the case of y^*_{it}, p^*_{it}, e^*_{it}, and m^*_{it} and capital flows in the case of equity price indices and interest rates, q^*_{it} and ρ^*_{it}. Glick and Rose (1999) provided a discussion of the importance of trade links in the analysis of contagion. It may also be desirable to allow for these weights to vary over time, to capture secular movements in the geographical patterns of trade and capital flows. However, too-frequent changes in the weights could introduce an undesirable degree of randomness into the analysis. This is the classic index number problem, for which a totally satisfactory answer does not exist. In our empirical analysis we use fixed trade weights but base their computation on averages of trade flows over a 3-year period. Specifically, w_{ij} can be measured as the total trade between country i and country j divided by the total trade of country i with *all* of its trading partners, where $w_{ii} = 0$ for all i.

It is worth noting that the exchange rate variable, e^*_{it}, defined for country i is not the same as the more familiar concept of the "effective exchange rate" as defined later. To see this, denote the exchange rate of country i in terms of the currency of country j by E_{ijt}. Then

$$\ln(E_{ijt}) = \ln(E_{it}/E_{jt}) = e_{it} - e_{jt}. \tag{5}$$

Let the trade share of country i with respect to country j be w_{ij}, and write the (log) effective exchange rate of country i as (recall that $e_{0t} = 0$)

$$\tilde{e}_{it} = \sum_{j=0}^{N} w_{ij}(e_{it} - e_{jt}) = \left(\sum_{j=0}^{N} w_{ij}\right) e_{it} - \sum_{j=1}^{N} w_{ij} e_{jt}.$$

But because $\sum_{j=0}^{N} w_{ij} = 1$, we have $\tilde{e}_{it} = e_{it} - \sum_{j=1}^{N} w_{ij} e_{jt}$, and hence $e^*_{it} = e_{it} - \tilde{e}_{it}$. Only in the case of the base country where $e_{0t} = 0$ do the two concepts (apart from a sign convention) coincide, namely we have $e^*_{0t} = -\tilde{e}_{0t}$.

Finally, in the case of countries or regions that attempt to maintain (approximately) a fixed effective exchange rate by pegging their currency to a basket of currencies, there will be a close correlation between e_{it} and e^*_{it}. Hence for the purposes of econometric analysis, it may not be advisable to include e^*_{it} as an exogenous variable in $\mathbf{x}^*_{it}$, considering that e_{it} is already included among the endogenous domestic variables. The inclusion of e_{it} in the model ought to be sufficient to accommodate the possible effects of exchange rate variations on the domestic economy. For the base economy, however, under our setup, e^*_{0t} will be determined by the models for the rest of the world via (1), for $i = 1, 2, \dots, N$. Hence for internal consistency, e^*_{0t} must be treated as an exogenous variable in the model for the base economy. Otherwise, there will be two sets of equations explaining e^*_{0t}, one equation derived by combining the exchange rate equations from the models for the regions $i = 1, 2, \dots, N$ and a second equation obtained directly from the model of country $i = 0$ if e^*_{0t} is included in that model as endogenous.

In general, the GVAR model allows for interactions among the different economies through three separate but interrelated channels:

1. Contemporaneous dependence of $\mathbf{x}_{it}$ on $\mathbf{x}^*_{it}$ and on its lagged values.
2. Dependence of the country-specific variables on common global exogenous variables, such as oil prices (see Sec. 5).
3. Nonzero contemporaneous dependence of shocks in country i on the shocks in country j, measured via the cross-country covariances, $\boldsymbol{\Sigma}_{ij}$,

$$\boldsymbol{\Sigma}_{ij} = \operatorname{cov}(\boldsymbol{\varepsilon}_{it}, \boldsymbol{\varepsilon}_{jt}) = E(\boldsymbol{\varepsilon}_{it}\boldsymbol{\varepsilon}'_{jt}) \qquad \text{for } i \neq j, \qquad (6)$$

   where $\boldsymbol{\varepsilon}_{it}$ is defined by (1). A typical element of $\boldsymbol{\Sigma}_{ij}$ will be denoted by $\sigma_{ij,\ell s} = \operatorname{cov}(\varepsilon_{i\ell t}, \varepsilon_{jst})$, which is the covariance of the ℓth variable in country i with the sth variable in country j.

The $N + 1$ country-specific models (1), together with the relations linking the (weakly) exogenous variables of the country-specific models to the variables in the rest of the global model, (4), provide a complete system. As emphasized in Section 1, due to data limitations for even moderate values of N, a full system estimation of the global model may not be feasible. To avoid this difficulty, we propose estimating the parameters of the country-specific models separately, treating the foreign-specific variables as weakly exogenous on the grounds that most economies (with the possible exception of the U.S.) are small relative to the size of the world economy. This is the standard assumption in the small open-economy macroeconomic literature pioneered by Fleming (1962) and Mundell (1963) and developed further by Dornbusch (1976), where it is routinely assumed that "world" interest rate, output, and prices are exogenously given. Whether such exogeneity assumptions hold in practice depends on the relative sizes of the countries/regions in the global model and on the degree of cross-country dependence of the idiosyncratic shocks, $\boldsymbol{\varepsilon}_{it}$, as captured by the cross-covariances $\boldsymbol{\Sigma}_{ij}$. Sufficient conditions under which foreign-specific variables can be viewed as weakly exogenous (or long-run forcing) in the context of the GVAR model are discussed in Section 7. Empirical evidence on the weak exogeneity of these variables is provided in Section 9.5.

3. SOLUTION OF THE GVAR MODEL

Due to the contemporaneous dependence of the domestic variables, $\mathbf{x}_{it}$, on the foreign variables, $\mathbf{x}^*_{it}$, the country-specific VAR models (1) need to be solved simultaneously for all of the domestic variables, $\mathbf{x}_{it}$, $i = 0, 1, \dots, N$. The solution can then be used for a variety of purposes, including forecasting, impulse response analysis, and risk management.

To construct the GVAR model from the country-specific models, we first define the $(k_i + k^*_i) \times 1$ vector

$$\mathbf{z}_{it} = \begin{pmatrix} \mathbf{x}_{it} \\ \mathbf{x}^*_{it} \end{pmatrix}, \qquad (7)$$

and then rewrite (1) as

$$\mathbf{A}_i \mathbf{z}_{it} = \mathbf{a}_{i0} + \mathbf{a}_{i1} t + \mathbf{B}_i \mathbf{z}_{i,t-1} + \boldsymbol{\varepsilon}_{it}, \qquad (8)$$

where

$$\mathbf{A}_i = (\mathbf{I}_{k_i}, -\boldsymbol{\Lambda}_{i0}) \qquad \text{and} \qquad \mathbf{B}_i = (\boldsymbol{\Phi}_i, \boldsymbol{\Lambda}_{i1}). \qquad (9)$$

The dimensions of $\mathbf{A}_i$ and $\mathbf{B}_i$ are $k_i \times (k_i + k^*_i)$, and $\mathbf{A}_i$ has a full row rank, namely $rank(\mathbf{A}_i) = k_i$.

Collect all of the country-specific variables together in the $k \times 1$ global vector $\mathbf{x}_t = (\mathbf{x}'_{0t}, \mathbf{x}'_{1t}, \dots, \mathbf{x}'_{Nt})'$, where $k = \sum_{i=0}^{N} k_i$ is the total number of the endogenous variables in the global model. Recall that $\mathbf{x}_{0t} = (y_{0t}, p_{0t}, q_{0t}, \rho_{0t}, m_{0t})'$ and $\mathbf{x}_{it} = (y_{it}, p_{it}, q_{it}, e_{it}, \rho_{it}, m_{it})'$ for $i = 1, 2, \dots, N$. Our analysis is invariant to the way in which the endogenous variables are stacked in $\mathbf{x}_{it}$ and to the ordering of the countries in $\mathbf{x}_t$.

It is now easily seen that the country-specific variables can all be written in terms of $\mathbf{x}_t$,

$$\mathbf{z}_{it} = \mathbf{W}_i \mathbf{x}_t, \qquad i = 0, 1, 2, \dots, N, \qquad (10)$$

where $\mathbf{W}_i$ is a $(k_i + k^*_i) \times k$ matrix of fixed (known) constants defined in terms of the country-specific weights w^y_{ij}, w^p_{ij}, w^q_{ij}, w^e_{ij}, w^ρ_{ij}, and w^m_{ij}. $\mathbf{W}_i$ can be viewed as the "link" matrix that allows the country-specific models to be written in terms of the global variable vector, $\mathbf{x}_t$.

Using (10) in (8), we have

$$\mathbf{A}_i \mathbf{W}_i \mathbf{x}_t = \mathbf{a}_{i0} + \mathbf{a}_{i1} t + \mathbf{B}_i \mathbf{W}_i \mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_{it},$$

where $\mathbf{A}_i \mathbf{W}_i$ and $\mathbf{B}_i \mathbf{W}_i$ are both $k_i \times k$-dimensional matrices. Stacking these equations now yields

$$\mathbf{G}\mathbf{x}_t = \mathbf{a}_0 + \mathbf{a}_1 t + \mathbf{H}\mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t, \qquad (11)$$

where

$$\mathbf{a}_0 = \begin{pmatrix} \mathbf{a}_{00} \\ \mathbf{a}_{10} \\ \vdots \\ \mathbf{a}_{N0} \end{pmatrix}, \qquad \mathbf{a}_1 = \begin{pmatrix} \mathbf{a}_{01} \\ \mathbf{a}_{11} \\ \vdots \\ \mathbf{a}_{N1} \end{pmatrix}, \qquad \boldsymbol{\varepsilon}_t = \begin{pmatrix} \boldsymbol{\varepsilon}_{0t} \\ \boldsymbol{\varepsilon}_{1t} \\ \vdots \\ \boldsymbol{\varepsilon}_{Nt} \end{pmatrix} \qquad (12)$$

and

$$\mathbf{G} = \begin{pmatrix} \mathbf{A}_0 \mathbf{W}_0 \\ \mathbf{A}_1 \mathbf{W}_1 \\ \vdots \\ \mathbf{A}_N \mathbf{W}_N \end{pmatrix}, \qquad \mathbf{H} = \begin{pmatrix} \mathbf{B}_0 \mathbf{W}_0 \\ \mathbf{B}_1 \mathbf{W}_1 \\ \vdots \\ \mathbf{B}_N \mathbf{W}_N \end{pmatrix}. \qquad (13)$$

It is easily seen that $\mathbf{G}$ is a $k \times k$-dimensional matrix and in general will be of full rank and hence nonsingular. Then the GVAR model in all of the variables can be written as

$$\mathbf{x}_t = \mathbf{G}^{-1}\mathbf{a}_0 + \mathbf{G}^{-1}\mathbf{a}_1 t + \mathbf{G}^{-1}\mathbf{H}\mathbf{x}_{t-1} + \mathbf{G}^{-1}\boldsymbol{\varepsilon}_t,$$

which may also be solved recursively forward to obtain the future values of $\mathbf{x}_t$; see Section 5 for further details.

It is worth illustrating this solution technique with a simple example. Consider a global model composed of three regions in three variables, say output, prices, and exchange rates (all in logs). Then

$$\mathbf{x}_t = \begin{pmatrix} \mathbf{x}_{0t} \\ \mathbf{x}_{1t} \\ \mathbf{x}_{2t} \end{pmatrix} = \begin{pmatrix} y_{0t} \\ p_{0t} \\ y_{1t} \\ p_{1t} \\ e_{1t} \\ y_{2t} \\ p_{2t} \\ e_{2t} \end{pmatrix}, \qquad \mathbf{z}_{0t} = \begin{pmatrix} y_{0t} \\ p_{0t} \\ y_{0t}^* \\ p_{0t}^* \\ e_{0t}^* \end{pmatrix},$$

and

$$\mathbf{z}_{it} = \begin{pmatrix} y_{it} \\ p_{it} \\ e_{it} \\ y_{it}^* \\ p_{it}^* \end{pmatrix}, \qquad i = 1, 2.$$

Using the trade shares, denoted simply by w_{ij}, to construct the foreign variables and noting that $e_{0t}^* = w_{01}e_{1t} + w_{02}e_{2t}$, the link matrices for these three regions are

$$\mathbf{W}_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & w_{01} & 0 & 0 & w_{02} & 0 & 0 \\ 0 & 0 & 0 & w_{01} & 0 & 0 & w_{02} & 0 \\ 0 & 0 & 0 & 0 & w_{01} & 0 & 0 & w_{02} \end{pmatrix} = \begin{pmatrix} \mathbf{I}_2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & w_{01}\mathbf{I}_3 & w_{02}\mathbf{I}_3 \end{pmatrix},$$

$$\mathbf{W}_1 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ w_{10} & 0 & 0 & 0 & 0 & w_{12} & 0 & 0 \\ 0 & w_{10} & 0 & 0 & 0 & 0 & w_{12} & 0 \end{pmatrix} = \begin{pmatrix} \mathbf{0} & \mathbf{I}_3 & \mathbf{0} & \mathbf{0} \\ w_{10}\mathbf{I}_2 & \mathbf{0} & w_{12}\mathbf{I}_2 & \mathbf{0} \end{pmatrix},$$

and

$$\mathbf{W}_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ w_{20} & 0 & w_{21} & 0 & 0 & 0 & 0 & 0 \\ 0 & w_{20} & 0 & w_{21} & 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_3 \\ w_{10}\mathbf{I}_2 & w_{12}\mathbf{I}_2 & \mathbf{0} & \mathbf{0} \end{pmatrix}.$$

Notice that the country-specific weights are nonnegative and satisfy the adding-up restrictions $w_{01} + w_{02} = 1$, $w_{10} + w_{12} = 1$, and $w_{20} + w_{21} = 1$. Furthermore, in the case where trade shares are nonzero, it is easily seen that the link matrices are of full row ranks, a property that will be of importance when we come to consider the error-correction properties of the global model in the following section. Finally,

$$\mathbf{A}_0 = (\mathbf{I}_2, -\boldsymbol{\Lambda}_{00}), \qquad \mathbf{A}_1 = (\mathbf{I}_3, -\boldsymbol{\Lambda}_{10}),$$

and

$$\mathbf{A}_2 = (\mathbf{I}_3, -\boldsymbol{\Lambda}_{20}),$$

where $\mathbf{I}_s$ is an identity matrix of order s. Using the foregoing $\mathbf{W}_i$ and $\mathbf{A}_i$ matrices, the $\mathbf{G}$ matrix defined by (13) can now be readily constructed. In this example, $\mathbf{G}$ is an 8×8 matrix and must be nonsingular if the global model is to be complete. A model is said to be complete if it is possible to uniquely solve for all of its endogenous variables.

4. ERROR–CORRECTING AND TRENDING PROPERTIES OF THE GLOBAL MODEL

It would be interesting to relate the error-correcting and trending properties of the country-specific models to those of the associated global model. The error-correction representation of (1) is given by

$$\Delta\mathbf{x}_{it} = \mathbf{a}_{i0} + \mathbf{a}_{i1}t - (\mathbf{I}_{k_i} - \boldsymbol{\Phi}_i)\mathbf{x}_{i,t-1} + (\boldsymbol{\Lambda}_{i0} + \boldsymbol{\Lambda}_{i1})\mathbf{x}_{i,t-1}^* + \boldsymbol{\Lambda}_{i0}\Delta\mathbf{x}_{it}^* + \boldsymbol{\varepsilon}_{it}, \qquad i = 0, 1, \ldots, N, \quad (14)$$

and, using (7),

$$\Delta\mathbf{x}_{it} = \mathbf{a}_{i0} + \mathbf{a}_{i1}t - (\mathbf{A}_i - \mathbf{B}_i)\mathbf{z}_{i,t-1} + \boldsymbol{\Lambda}_{i0}\Delta\mathbf{x}_{it}^* + \boldsymbol{\varepsilon}_{it}, \quad (15)$$

where, as before, $\mathbf{z}_{it} = (\mathbf{x}_{it}', \mathbf{x}_{it}^{*\prime})'$ and $\mathbf{A}_i$ and $\mathbf{B}_i$ are already defined by (9). The error-correction properties of the model for country/region i are summarized in the $k_i \times (k_i + k_i^*)$ matrix

$$\boldsymbol{\Pi}_i = \mathbf{A}_i - \mathbf{B}_i. \quad (16)$$

In particular, the rank of $\boldsymbol{\Pi}_i$, say $r_i \leq k_i$, specifies the number of "long-run" relationships that exist among the domestic and the country-specific foreign variables, namely $\mathbf{x}_{it}$ and $\mathbf{x}_{it}^*$. Therefore, we have

$$\mathbf{A}_i - \mathbf{B}_i = \boldsymbol{\alpha}_i\boldsymbol{\beta}_i', \quad (17)$$

where $\boldsymbol{\alpha}_i$ is the $k_i \times r_i$ loading matrix of full column rank and $\boldsymbol{\beta}_i$ is the $(k_i + k_i^*) \times r_i$ matrix of cointegrating vectors, also of full column rank.

In the case where $\boldsymbol{\Pi}_i$ is rank deficient and the linear trend coefficients, $\mathbf{a}_{i1}$, are unrestricted, the linear trend in the error-correction model transforms into a quadratic trend in $\mathbf{x}_{it}$, which is clearly undesirable. It would be more appropriate to retain the same deterministic trend properties for the elements of $\mathbf{x}_{it}$ under different rank restrictions on $\boldsymbol{\Pi}_i$. As was shown by, for example, Pesaran, Shin, and Smith (2000), this can be achieved by restricting the trend coefficients so that

$$\mathbf{a}_{i1} = (\mathbf{A}_i - \mathbf{B}_i)\boldsymbol{\kappa}_i, \quad (18)$$

where $\boldsymbol{\kappa}_i$ is a $(k_i + k_i^*) \times 1$ vector of fixed constants. This specification imposes $k_i - r_i$ restrictions on the trend coefficients.

Consider now the global model given by (11), which has the error-correction form

$$\mathbf{G}\Delta\mathbf{x}_t = \mathbf{a}_0 + \mathbf{a}_1 t - (\mathbf{G} - \mathbf{H})\mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t. \quad (19)$$

The number of long-run relationships in the global model is similarly determined by the rank of $\mathbf{G}-\mathbf{H}$. Using (13) and (17), we first note that

$$\mathbf{G}-\mathbf{H}=\begin{pmatrix}(\mathbf{A}_0-\mathbf{B}_0)\mathbf{W}_0\\(\mathbf{A}_1-\mathbf{B}_1)\mathbf{W}_1\\\vdots\\(\mathbf{A}_N-\mathbf{B}_N)\mathbf{W}_N\end{pmatrix}=\begin{pmatrix}\boldsymbol{\alpha}_0\boldsymbol{\beta}_0'\mathbf{W}_0\\\boldsymbol{\alpha}_1\boldsymbol{\beta}_1'\mathbf{W}_1\\\vdots\\\boldsymbol{\alpha}_N\boldsymbol{\beta}_N'\mathbf{W}_N\end{pmatrix},$$

which can be written equivalently as

$$\mathbf{G}-\mathbf{H}=\tilde{\boldsymbol{\alpha}}\tilde{\boldsymbol{\beta}}',$$

where $\tilde{\boldsymbol{\alpha}}$ is the $k\times r$ block-diagonal matrix of the global loading coefficients

$$\tilde{\boldsymbol{\alpha}}=\begin{pmatrix}\boldsymbol{\alpha}_0&\mathbf{0}&\dots&\mathbf{0}\\\mathbf{0}&\boldsymbol{\alpha}_1&\dots&\mathbf{0}\\\vdots&\vdots&\ddots&\vdots\\\mathbf{0}&\mathbf{0}&\dots&\boldsymbol{\alpha}_N\end{pmatrix},\tag{20}$$

$$\tilde{\boldsymbol{\beta}}=(\mathbf{W}_0'\boldsymbol{\beta}_0,\mathbf{W}_1'\boldsymbol{\beta}_1,\dots,\mathbf{W}_N'\boldsymbol{\beta}_N),\tag{21}$$

$r=\sum_{i=0}^N r_i$, and $k=\sum_{i=0}^N k_i$. It is clear that $rank(\tilde{\boldsymbol{\alpha}})=\sum_{i=0}^N rank(\boldsymbol{\alpha}_i)=r$.

Consider now the global $k\times r$ cointegrating matrix $\tilde{\boldsymbol{\beta}}$. Each of the blocks in $\tilde{\boldsymbol{\beta}}$, namely $\mathbf{W}_i'\boldsymbol{\beta}_i$, are of dimension $k\times r_i$ with rank at most equal to r_i. Therefore, the rank of $\tilde{\boldsymbol{\beta}}$ will be at most equal to r. Namely, *the number of the long-run relationships in the global model cannot exceed the sum of the numbers of long-run relations that exist in the country/region-specific models.* It is worth noting that this result is conditional on the choice of the link matrices, $\mathbf{W}_i$, and in principle it would be possible to obtain a different number of cointegrating relations in the global model for different choices of the link matrices.

The deterministic trend properties of the GVAR model are also related to those of the underlying country-specific models. As with the country-specific models, to ensure that rank restrictions on $\mathbf{G}-\mathbf{H}$ do not lead to quadratic trends in the variables of the global model, the vector of trend coefficients, $\mathbf{a}_1$, must satisfy the restrictions

$$\mathbf{a}_1=(\mathbf{G}-\mathbf{H})\boldsymbol{\gamma},$$

where $\boldsymbol{\gamma}$ is a $k\times 1$ vector of fixed constants. Therefore, for the deterministic trend properties of the variables to be the same in the global model as in the underlying country-specific models, using (18), we must have

$$(\mathbf{G}-\mathbf{H})\boldsymbol{\gamma}=\begin{pmatrix}(\mathbf{A}_0-\mathbf{B}_0)\boldsymbol{\kappa}_0\\(\mathbf{A}_1-\mathbf{B}_1)\boldsymbol{\kappa}_1\\\vdots\\(\mathbf{A}_N-\mathbf{B}_N)\boldsymbol{\kappa}_N\end{pmatrix}.$$

This condition is satisfied if

$$\boldsymbol{\kappa}_i=\mathbf{W}_i\boldsymbol{\gamma}\qquad\text{for } i=0,1,\dots,N.$$

These impose additional cross-country restrictions on the trend coefficients. Altough in principal it should be possible to test these restrictions, their simultaneous imposition will be infeasible when N is large compared with the available time series data, T.

5. DYNAMIC PROPERTIES, STABILITY CONDITIONS, AND FORECASTS OF THE GVAR MODEL

In this section we consider the dynamic properties of a slightly generalized version of the global model that allows for "common global variables" such as oil prices. Such an augmented VARX* model is given by

$$\mathbf{x}_{it}=\mathbf{a}_{i0}+\mathbf{a}_{i1}t+\boldsymbol{\Phi}_i\mathbf{x}_{i,t-1}+\boldsymbol{\Lambda}_{i0}\mathbf{x}_{it}^*+\boldsymbol{\Lambda}_{i1}\mathbf{x}_{i,t-1}^*+\boldsymbol{\Psi}_{i0}\mathbf{d}_t+\boldsymbol{\Psi}_{i1}\mathbf{d}_{t-1}+\boldsymbol{\varepsilon}_{it},\tag{22}$$

for $t=1,2,\dots,T$ and $i=0,1,2,\dots,N$, where $\mathbf{d}_t$ is an $s\times 1$ vector of common global variables assumed to be weakly exogenous to the global economy. The distinction between foreign variables, $\mathbf{x}_{it}^*$, and the global exogenous variables, $\mathbf{d}_t$, is relevant for the analysis of the dynamic properties of the global model but is not of material consequence for estimation of the country-specific models. For the latter purpose, $\mathbf{x}_{it}^*$ and $\mathbf{d}_t$ can be combined and treated jointly as weakly exogenous.

The global model associated with these country-specific models is now given by

$$\mathbf{G}\mathbf{x}_t=\mathbf{a}_0+\mathbf{a}_1t+\mathbf{H}\mathbf{x}_{t-1}+\boldsymbol{\Psi}_0\mathbf{d}_t+\boldsymbol{\Psi}_1\mathbf{d}_{t-1}+\boldsymbol{\varepsilon}_t,$$

where $\mathbf{a}_0$, $\mathbf{a}_1$, $\mathbf{G}$, $\mathbf{H}$, and $\boldsymbol{\varepsilon}_t$ are as already defined by (12) and (13) and

$$\boldsymbol{\Psi}_0=\begin{pmatrix}\boldsymbol{\Psi}_{00}\\\boldsymbol{\Psi}_{10}\\\vdots\\\boldsymbol{\Psi}_{N0}\end{pmatrix}\quad\text{and}\quad\boldsymbol{\Psi}_1=\begin{pmatrix}\boldsymbol{\Psi}_{01}\\\boldsymbol{\Psi}_{11}\\\vdots\\\boldsymbol{\Psi}_{N1}\end{pmatrix}.\tag{23}$$

Assuming that $\mathbf{G}$ is nonsingular, we now have the reduced-form global model

$$\mathbf{x}_t=\mathbf{b}_0+\mathbf{b}_1t+\boldsymbol{F}\mathbf{x}_{t-1}+\boldsymbol{\Upsilon}_0\mathbf{d}_t+\boldsymbol{\Upsilon}_1\mathbf{d}_{t-1}+\mathbf{u}_t,\tag{24}$$

for $t=1,2,\dots,T,T+1,\dots,T+n$, where

$$\begin{aligned}&\mathbf{b}_i=\mathbf{G}^{-1}\mathbf{a}_i,\quad i=0,1;\qquad \boldsymbol{F}=\mathbf{G}^{-1}\mathbf{H},\\&\boldsymbol{\Upsilon}_0=\mathbf{G}^{-1}\boldsymbol{\Psi}_0,\qquad\boldsymbol{\Upsilon}_1=\mathbf{G}^{-1}\boldsymbol{\Psi}_1,\qquad\text{and}\qquad\mathbf{u}_t=\mathbf{G}^{-1}\boldsymbol{\varepsilon}_t.\end{aligned}\tag{25}$$

Suppose now that the global economy is observed over the period $t=1,2,\dots,T$, and that we wish to forecast $\mathbf{x}_t$ over the future periods $t=T+1,T+2,\dots,T+n$, where n is the forecast horizon. To simplify the exposition, we assume that the exogenous variables $\mathbf{d}_t$ for $t=T+1,T+2,\dots$ are given. But the analysis can be easily generalized to allow for the uncertainty of the exogenous global variables. We do this in Section 10 when we discuss the effect of shocks on the loss distribution, which is a nonlinear function of the shocks. But for impulse response analysis, due to the linearity of the underlying relationships, the impulse response functions do not depend on the processes generating the global variables when they are strictly exogenous.

For given values of $\mathbf{d}_t$, $t=T+1,T+2,\dots$, and solving the difference equation (24) forward, we obtain

$$\begin{aligned}\mathbf{x}_{T+n}=\boldsymbol{F}^n\mathbf{x}_T&+\sum_{\tau=0}^{n-1}\boldsymbol{F}^\tau[\mathbf{b}_0+\mathbf{b}_1(T+n-\tau)]\\&+\sum_{\tau=0}^{n-1}\boldsymbol{F}^\tau[\boldsymbol{\Upsilon}_0\mathbf{d}_{T+n-\tau}+\boldsymbol{\Upsilon}_1\mathbf{d}_{T+n-\tau-1}]\\&+\sum_{\tau=0}^{n-1}\boldsymbol{F}^\tau\mathbf{u}_{T+n-\tau}.\end{aligned}\tag{26}$$

This solution has four distinct components. The first component, $\mathcal{F}^n\mathbf{x}_T$, measures the effect of initial values, $\mathbf{x}_T$, on the future state of the system. The second component captures the deterministic trends embodied in the underlying VAR model. The third component measures the effect of the global exogenous variables, $\mathbf{d}_t$, on the model's endogenous variables, $\mathbf{x}_t$. Finally, the last term in (26) represents the stochastic (unpredictable) component of $\mathbf{x}_{T+n}$. The point forecasts of the endogenous variables conditional on the initial state of the system and the exogenous global variables are now given by

$$\mathbf{x}^*_{T+n} = E\left(\mathbf{x}_{T+n}\middle|\mathbf{x}_T, \bigcup_{\tau=1}^{n}\mathbf{d}_{T+\tau}\right)$$
$$= \mathcal{F}^n\mathbf{x}_T + \sum_{\tau=0}^{n-1}\mathcal{F}^\tau[\mathbf{b}_0 + \mathbf{b}_1(T+n-\tau)]$$
$$+ \sum_{\tau=0}^{n-1}\mathcal{F}^\tau[\boldsymbol{\Upsilon}_0\mathbf{d}_{T+n-\tau} + \boldsymbol{\Upsilon}_1\mathbf{d}_{T+n-\tau-1}]. \quad (27)$$

The probability distribution function of $\mathbf{x}_{T+n}$, needed for the computation of the loss distribution of a given portfolio, can also be obtained under suitable assumptions concerning the probability distribution function of the shocks, $\boldsymbol{\varepsilon}_t$. Under the assumption that $\boldsymbol{\varepsilon}_t$ is normally distributed, we have

$$\mathbf{x}_{T+n}\Big|\mathbf{x}_T, \bigcup_{\tau=1}^{n}\mathbf{d}_{T+\tau} \backsim \mathrm{N}(\mathbf{x}^*_{T+n}, \boldsymbol{\Omega}_n), \quad (28)$$

where $\mathbf{x}^*_{T+n}$ is given by (27) and

$$\boldsymbol{\Omega}_n = \sum_{\tau=0}^{n-1}\mathcal{F}^\tau\mathbf{G}^{-1}\boldsymbol{\Sigma}\mathbf{G}'^{-1}\mathcal{F}'^\tau, \quad (29)$$

where $\boldsymbol{\Sigma}$ is the $k \times k$ variance-covariance matrix of the shocks, $\boldsymbol{\varepsilon}_t$. Note that the (i,j) block of $\boldsymbol{\Sigma}$ is given by $\boldsymbol{\Sigma}_{ij}$, which is defined by (6). Estimation of $\boldsymbol{\Sigma}_{ij}$ and the other parameters is addressed later.

The dynamic properties of the global model depend crucially on the eigenvalues of $\mathcal{F}$. In the trend-stationary case where all of the roots of $\mathcal{F}$ lie inside the unit circle, $\mathbf{x}_{T+n}$ will have a stable distribution and will satisfy the following properties:

- The dependence of $\mathbf{x}_{T+n}$ on the initial values, $\mathbf{x}_T$, will disappear for sufficiently large values of n, the forecast horizon.
- The forecast covariance matrix, $\boldsymbol{\Omega}_n$, will converge to a finite value as $n \to \infty$.
- The point forecasts, $\mathbf{x}^*_{T+n}$, will exhibit the same linear trending property as specified in the underlying country-specific VAR models.

In contrast, when one or more roots of $\mathcal{F}$ fall on the unit circle, none of the foregoing properties hold. The unit eigenvalues correspond to the unit roots and cointegrating properties of the various variables in the global VAR model:

- The multiplier matrix $\mathcal{F}^n$ converges to a nonzero matrix of fixed constants even if n is allowed to increase without bound, and the dependence of $\mathbf{x}^*_{T+n}$ on the initial values does not disappear as $n \to \infty$.
- The forecast covariance matrix, $\boldsymbol{\Omega}_n$, will rise linearly with n, indicating a steady deterioration in the precision with which values of $\mathbf{x}_{T+n}$ are forecast with the horizon, n.
- Finally, as noted in Section 4, the linear trend in the underlying VAR model when combined with a unit root in $\mathcal{F}$ generates a quadratic trend in the level of the variables.

Some of these undesirable features can be avoided or bypassed. For example, to avoid increasing forecast error variances, one could focus on forecasting growth rates (using the GVAR in levels). As noted earlier, quadratic trends can be eliminated by restricting the trend coefficients $\mathbf{b}_1$. Although imposing these restrictions exactly would not be feasible when N is large relative to T, a partial solution can be achieved by imposing the restrictions, (18), on the trend coefficients of the country-specific models. This estimation problem is feasible and is discussed in Section 7.

6. IMPULSE RESPONSE ANALYSIS

One of the important tools in the analysis of dynamic systems is the impulse response function, which characterizes the possible response of the system at different future periods to the effect of shocking one of the variables in the model. For example, it may be of interest to work out the effect of a shock of a given size to the yen/dollar exchange rate on the evolution of real output in Germany. In carrying out such an analysis, it is important that the correlations that exist across the different shocks, both within each country and across the different countries, are accounted for in an appropriate manner. In the traditional VAR literature, this is accomplished by means of the orthogonalized impulse responses (OIR) of Sims (1980), where impulse responses are computed with respect to a set of orthogonalized shocks, say $\boldsymbol{\xi}_t$, instead of the original shocks, $\boldsymbol{\varepsilon}_t$. The link between the two sets of shocks is given by $\boldsymbol{\xi}_t = \mathbf{P}^{-1}\boldsymbol{\varepsilon}_t$, where $\mathbf{P}$ is a $k \times k$ lower triangular Cholesky factor of the variance-covariance matrix, $\mathrm{cov}(\boldsymbol{\varepsilon}_t) = \boldsymbol{\Sigma}$, namely

$$\mathbf{PP}' = \boldsymbol{\Sigma}. \quad (30)$$

Therefore, by construction, $E(\boldsymbol{\xi}_t\boldsymbol{\xi}'_t) = \mathbf{I}_k$. The $k \times 1$ vector of the OIR function of a unit shock (equal to one standard error) to the jth equation on $\mathbf{x}_{t+n}$ is given by

$$\boldsymbol{\psi}^o_j(n) = \mathcal{F}^n\mathbf{G}^{-1}\mathbf{P}\mathfrak{s}_j, \qquad n = 0, 1, 2, \ldots, \quad (31)$$

where $\mathfrak{s}_j$ is a $k \times 1$ selection vector with unity as its jth element (corresponding to a particular shock in a particular country) and zeros elsewhere. In the case of the global VAR model, the OIRs also depend on the order of factors in each region/country and on the order in which the countries are stacked in $\mathbf{x}_t$. Mathematically, this non-invariance property of the OIRs is due simply to the non-uniqueness of the Cholesky factor, $\mathbf{P}$.

The OIR function is usually used for small systems that admit a natural causal ordering for the variables in the VAR. But in general such a natural ordering does not exist, and the OIR functions are not unique and sometimes depend critically on the order in which the variables are included in the VAR. The more recent literature emphasizes the use of "structural VAR" methodology to identify the shocks. This is achieved by imposing a priori restrictions on the covariance matrix of the shocks and/or on long-run impulse responses themselves. For example, the work of Bernanke (1986), Blanchard and Watson (1986), and Sims (1986) consider *a priori* restrictions on contemporaneous covariance matrix of shocks, and Blanchard and Quah (1989) and Clarida and Gali (1994) consider restrictions on the long-run impact of shocks to identify the impulse responses. Although such a strategy may be operational when the VAR contains only a few variables, its application to the GVAR model does not seem feasible. In the GVAR model with $N+1$ countries and k_i endogenous variables per country, exact identification of the shocks will require $\sum_{i=0}^{N} k_i(k_i-1)$ restrictions. For example, in the case of the model considered empirically in Section 9, we would need to motivate 300 different theory-based restrictions; it is not clear to us how this could be achieved.

An alternative approach that is invariant to the ordering of the variables and the countries in the global VAR would be to use (26) directly, shock only one element (say the jth shock in $\boldsymbol{\varepsilon}_t$, corresponding to the ℓth variable in the ith country) and integrate out the effects of other shocks using an assumed or the historically observed distribution of the errors. This approach has been advanced by Koop, Pesaran, and Potter (1996), Pesaran and Shin (1998), and Pesaran and Smith (1998) and yields the generalized impulse response function (GIRF),

$$\mathbf{GI}_{x:\varepsilon_{i\ell}}(n, \sqrt{\sigma_{ii,\ell\ell}}, \mathcal{I}_{t-1}) = E(\mathbf{x}_{t+n}|\varepsilon_{i\ell t} = \sqrt{\sigma_{ii,\ell\ell}}, \mathcal{I}_{t-1}) - E(\mathbf{x}_{t+n}|\mathcal{I}_{t-1}), \quad (32)$$

where $\mathcal{I}_t = (\mathbf{x}_t, \mathbf{x}_{t-1}, \ldots)$ is the information set at time $t-1$ and $\mathbf{d}_t$ is assumed to be given exogenously. On the assumption that $\boldsymbol{\varepsilon}_t$ has a multivariate normal distribution, and using (26), it is now easily seen that

$$\boldsymbol{\psi}_j^g(n) = \frac{1}{\sqrt{\sigma_{ii,\ell\ell}}} \boldsymbol{F}^n \mathbf{G}^{-1} \boldsymbol{\Sigma} \mathfrak{s}_j, \qquad n = 0, 1, 2, \ldots, \quad (33)$$

which measures the effect of one standard error shock to the jth equation (corresponding to the ℓth variable in the ith country) at time t on expected values of $\mathbf{x}$ at time $t+n$. $\boldsymbol{\psi}_j^g(n)$ will be identical to $\boldsymbol{\psi}_j^o(n)$ when $\boldsymbol{\Sigma}$ is diagonal or when the focus of the analysis is on the impulse response function of shocking the first element of $\boldsymbol{\varepsilon}_t$.

6.1 Impulse Response Analysis of Shocks to the Global Exogenous Variables

In this section we derive generalized impulse response functions for a unit shock to the ith exogenous variable, d_{it}. For this purpose, we need to specify a dynamic process for the exogenous variables. Suppose that $\mathbf{d}_t$ follows a first-order autoregressive process,

$$\mathbf{d}_t = \boldsymbol{\mu}_d + \boldsymbol{\Phi}_d \mathbf{d}_{t-1} + \boldsymbol{\varepsilon}_{dt}, \qquad \boldsymbol{\varepsilon}_{dt} \backsim \text{iid}(\mathbf{0}, \boldsymbol{\Sigma}_d), \quad (34)$$

where $\boldsymbol{\mu}_d$ is an $s \times 1$ vector of constants, $\boldsymbol{\Phi}_d$ is $s \times s$ matrix of lagged coefficients, $\boldsymbol{\varepsilon}_{dt}$ is an $s \times 1$ vector of shocks to the exogenous variables, and $\boldsymbol{\Sigma}_d$ is the covariance matrix of these shocks, which we allow to be singular. This allows for the possibility that some of the elements of $\mathbf{d}_t$ could be perfectly predictable (e.g., linear trends, deterministic seasonal effects). As before, the GIRF of the effect of a unit shock to the ith exogenous variable on the vector of the endogenous variables n periods ahead is defined by

$$\mathbf{GI}_{x:d_i}(n, \sigma_{d,ii}, \mathcal{I}_{t-1}) = E(\mathbf{x}_{t+n}|d_{it} = \sqrt{\sigma_{d,ii}}, \mathcal{I}_{t-1}) - E(\mathbf{x}_{t+n}|\mathcal{I}_{t-1}), \quad (35)$$

where $\sigma_{d,ii}$ is the ith diagonal element of $\boldsymbol{\Sigma}_d$. Using (24), it is now easily seen that

$$\begin{aligned}\mathbf{GI}_{x:d_i}(n, \sigma_{d,ii}, \mathcal{I}_{t-1}) &= \boldsymbol{F}\mathbf{GI}_{x:d_i}(n-1, \sigma_{d,ii}, \mathcal{I}_{t-1}) + \boldsymbol{\Upsilon}_0 \mathbf{GI}_{d:d_i}(n, \sigma_{d,ii}, \mathcal{I}_{t-1}) \\ &\quad + \boldsymbol{\Upsilon}_1 \mathbf{GI}_{d:d_i}(n-1, \sigma_{d,ii}, \mathcal{I}_{t-1})\end{aligned} \quad (36)$$

for $n = 0, 1, 2, \ldots$, where

$$\mathbf{GI}_{d:d_i}(n, \sigma_{d,ii}, \mathcal{I}_{t-1}) = E(\mathbf{d}_{t+n}|d_{it} = \sqrt{\sigma_{d,ii}}, \mathcal{I}_{t-1}) - E(\mathbf{d}_{t+n}|\mathcal{I}_{t-1}). \quad (37)$$

It is now easily seen that for $n < 1$, $\mathbf{GI}_{x:d_i}(n-1, \sigma_{d,ii}, \mathcal{I}_{t-1}) = \mathbf{GI}_{d:d_i}(n-1, \sigma_{d,ii}, \mathcal{I}_{t-1}) = 0$, and

$$\mathbf{GI}_{x:d_i}(0, \sigma_{d,ii}, \mathcal{I}_{t-1}) = \boldsymbol{\Upsilon}_0 \mathbf{GI}_{d:d_i}(0, \sigma_{d,ii}, \mathcal{I}_{t-1}).$$

Similarly,

$$\mathbf{GI}_{d:d_i}(0, \sigma_{d,ii}, \mathcal{I}_{t-1}) = \frac{1}{\sqrt{\sigma_{d,ii}}} \boldsymbol{\Sigma}_d \mathfrak{e}_i,$$

where $\mathfrak{e}_i$ is an $s \times 1$ selection vector with its ith element unity and other elements 0, and

$$\mathbf{GI}_{d:d_i}(n, \sigma_{d,ii}, \mathcal{I}_{t-1}) = \boldsymbol{\Phi}_d \mathbf{GI}_{d:d_i}(n-1, \sigma_{d,ii}, \mathcal{I}_{t-1}) \qquad \text{for } n = 1, 2, \ldots.$$

Hence

$$\mathbf{GI}_{d:d_i}(n, \sigma_{d,ii}, \mathcal{I}_{t-1}) = \frac{1}{\sqrt{\sigma_{d,ii}}} \boldsymbol{\Phi}_d^n \boldsymbol{\Sigma}_d \mathfrak{e}_i \qquad \text{for } n = 0, 1, \ldots.$$

Substituting this result in (36), we have

$$\begin{aligned}\mathbf{GI}_{x:d_i}(n, \sigma_{d,ii}, \mathcal{I}_{t-1}) &= \boldsymbol{F}\mathbf{GI}_{x:d_i}(n-1, \sigma_{d,ii}, \mathcal{I}_{t-1}) \\ &\quad + \frac{1}{\sqrt{\sigma_{d,ii}}} (\boldsymbol{\Upsilon}_0 \boldsymbol{\Phi}_d + \boldsymbol{\Upsilon}_1) \boldsymbol{\Phi}_d^{n-1} \boldsymbol{\Sigma}_d \mathfrak{e}_i\end{aligned} \quad (38)$$

for $n = 1, 2, \ldots$, where

$$\mathbf{GI}_{x:d_i}(0, \sigma_{d,ii}, \mathcal{I}_{t-1}) = \frac{1}{\sqrt{\sigma_{d,ii}}} \boldsymbol{\Upsilon}_0 \boldsymbol{\Sigma}_d \mathfrak{e}_i. \quad (39)$$

In the simple case where $\mathbf{d}$ is a scalar variable (such as oil price), $\frac{1}{\sqrt{\sigma_{d,ii}}} \boldsymbol{\Sigma}_d \mathfrak{e}_i = \sqrt{\sigma_{d,ii}}$.

7. ESTIMATION OF THE GVAR MODEL AS INDIVIDUAL PARTIAL SYSTEMS

As was pointed out earlier, a system estimation of the VAR model in (24) will not be feasible even for moderate values of N. The unconstrained estimation of (24) would involve estimating a large number of parameters often greater than the number of available observations! But the modeling approach set out earlier is feasible even for a relatively large number of countries/regions. This is due to the fact that the weights w_{ij}, $i, j = 0, 1, \ldots, N$, are not estimated simultaneously with the other country-specific parameters, but rather are computed from cross-country data on trade and/or capital flow accounts. Also, the estimation of the country-specific parameters is carried out on a country-by-country basis rather than simultaneously. This is justified if N is sufficiently large and the following conditions hold:

1. Stability. The global model, (24), formed from the country-specific models is dynamically stable; namely, the eigenvalues of matrix $\boldsymbol{F}$ defined by (25) are either on or inside the unit circle.
2. Smallness. The weights used in the construction of foreign-specific variables, $w_{ij} \geq 0$, are small, being of order $1/N$ such that
$$\sum_{j=0}^{N} w_{ij}^2 \to 0, \qquad \text{as } N \to \infty, \text{ for all } i.$$
3. Weak dependence. The cross-dependence of the idiosyncratic shocks, if any, is sufficiently small so that
$$\frac{\sum_{j=0}^{N} \sigma_{ij,ls}}{N} \to 0, \qquad \text{as } N \to \infty, \text{ for all } i, l, \text{ and } s,$$
where $\sigma_{ij,\ell s} = \operatorname{cov}(\varepsilon_{i\ell t}, \varepsilon_{jst})$ is the covariance of the ℓth variable in country i with the sth variable in country j.

These conditions are sufficient for $\operatorname{cov}(\mathbf{x}_{it}^*, \boldsymbol{\varepsilon}_{it}) \to 0$ as $N \to \infty$. They provide a formalization of the concept of "small open economy" from the perspective of econometric analysis. The need for conditions 1 and 2 is rather obvious. Clearly, condition 3 is satisfied when the country-specific shocks are purely idiosyncratic; but it is also satisfied for certain degree of dependence across the idiosyncratic shocks. For example, the condition is met if there exists an ordering (j), seen from the viewpoint of country i, for which $\sigma_{i(j),ls}$ decays exponentially with $|i - (j)|$. It is not necessary for this ordering to be known, and it need not be the same for other countries/regions. In this sense, condition 3 allows for the idiosyncratic shocks to be "weakly correlated."

In practice, however, it would not be possible to check the validity of these conditions directly. But, as shown in Section 7.1, the implications of the weak exogeneity condition can be tested indirectly. Under weak exogeneity, the parameters of the country-specific models can be estimated consistently by the ordinary least squares (OLS) approach or by the reduced-rank procedure applied directly to (22). OLS estimation is clearly much simpler but suffers from the shortcoming that it does not fully allow for the fact that one or more of the six factors used in the model may have unit roots, and does not take into account the important possibility that the level of domestic and foreign variables may be tied together in the long run (the phenomenon known as "cointegration" in the econometric literature). To deal with the unit root problem, many researchers in the past have estimated the VAR model in first differences (using rates of changes of the factors rather than their logarithms). But the first-differencing operation can be inefficient when there are in fact cointegrating relations among the factors and can be avoided by the reduced-rank regression approach.

The reduced-rank estimation procedure in the case where all of the variables in the model are treated as endogenous $I(1)$ has been developed by Johansen (1988, 1995). But in the context of the GVAR model (1) for estimation purposes, the foreign variables, $\mathbf{x}_{it}^*$, are treated as exogenous, and Johansen's approach must be modified to take this into account. Appropriate methods for estimating reduced-rank regressions containing weakly exogenous $I(1)$ regressors have been developed by Harbo, Johansen, Nielsen, and Rahbek (1998) and Pesaran et al. (2000). The concept of weak exogeneity in a system of $I(1)$ variables is also closely related to the notions of "long-run causality" and "long-run forcing" discussed by Granger and Lin (1995) and Pesaran et al. (2000). Here we provide some basic background to motivate the identification of the error-correction terms and the weak exogeneity test, which is discussed in Section 7.1.

To estimate the country-specific models subject to reduced-rank restrictions, first the error-correction equation (22) is rewritten as

$$\Delta \mathbf{x}_{it} = \mathbf{a}_{i0} + \mathbf{a}_{i1} t - \boldsymbol{\Pi}_i \mathbf{v}_{i,t-1} + \boldsymbol{\Lambda}_{i0} \Delta \mathbf{x}_{it}^* + \boldsymbol{\Psi}_{i0} \Delta \mathbf{d}_t + \boldsymbol{\varepsilon}_{it}, \quad (40)$$

where

$$\boldsymbol{\Pi}_i = (\mathbf{A}_i - \mathbf{B}_i, -\boldsymbol{\Psi}_{i0} - \boldsymbol{\Psi}_{i1}) \quad (41)$$

and

$$\mathbf{v}_{i,t-1} = \begin{pmatrix} \mathbf{z}_{i,t-1} \\ \mathbf{d}_{t-1} \end{pmatrix}. \quad (42)$$

To avoid the problem of introducing quadratic trends in the level of the variables when $\boldsymbol{\Pi}_i$ is rank deficient as before, we impose the restrictions $\mathbf{a}_{i1} = \boldsymbol{\Pi}_i \boldsymbol{\kappa}_i$, which reduce to (18) when there are no global exogenous variables in the model; the dimension of $\boldsymbol{\kappa}_i$ is now $(k_i + k_i^* + s) \times 1$. Under these restrictions, (40) becomes

$$\Delta \mathbf{x}_{it} = \mathbf{c}_{i0} - \boldsymbol{\Pi}_i [\mathbf{v}_{i,t-1} - \boldsymbol{\kappa}_i (t-1)] + \boldsymbol{\Lambda}_{i0} \Delta \mathbf{x}_{it}^* + \boldsymbol{\Psi}_{i0} \Delta \mathbf{d}_t + \boldsymbol{\varepsilon}_{it}, \quad (43)$$

where

$$\mathbf{c}_{i0} = \mathbf{a}_{i0} + \boldsymbol{\Pi}_i \boldsymbol{\kappa}_i, \quad (44)$$

where $\boldsymbol{\Pi}_i$ is a $k_i \times (k_i + k_i^* + s)$ matrix that provides information on the long-run–level relationships that may exist among the model's variables. In the case where *all* of the variables, $\mathbf{z}_{it}$ and $\mathbf{d}_t$, are $I(1)$ and are not cointegrated, $\boldsymbol{\Pi}_i$ is equal to 0 and (43) reduces to the first-differenced model

$$\Delta \mathbf{x}_{it} = \mathbf{a}_{i0} + \boldsymbol{\Lambda}_{i0} \Delta \mathbf{x}_{it}^* + \boldsymbol{\Psi}_{i0} \Delta \mathbf{d}_t + \boldsymbol{\varepsilon}_{it}. \quad (45)$$

It is interesting to note that this specification leads to random-walk models (augmented by oil price changes) for the global variables, $\mathbf{z}_t$. Using the solution technique of Section 3, we have

$$\mathbf{G} \Delta \mathbf{z}_t = \mathbf{a}_0 + \boldsymbol{\Psi}_0 \Delta \mathbf{d}_t + \boldsymbol{\varepsilon}_t$$

or

$$\Delta \mathbf{z}_t = \mathbf{G}^{-1}\mathbf{a}_0 + \mathbf{G}^{-1}\boldsymbol{\Psi}_0 \Delta \mathbf{d}_t + \mathbf{G}^{-1}\boldsymbol{\varepsilon}_t,$$

where $\mathbf{G}$ and $\boldsymbol{\Psi}_0$ are defined by (13) and (23). Therefore, as anticipated by the analysis of Section 4, there will be no long-run relationship in the global model if there are no long-run relationships in the underlying regional models.

But in general, due to long-term interlinkages that exist between domestic and foreign variables as well as between the domestic variables themselves, one would expect $\boldsymbol{\Pi}_i$ to be nonzero but rank deficient. The rank of $\boldsymbol{\Pi}_i$ identifies the number of long-run or cointegrating relationships. Rank deficiency arises when $rank(\boldsymbol{\Pi}_i) = r_i$ and $r_i < k_i$. In the more general case where $\boldsymbol{\Pi}_i$ is nonzero but could (possibly) be rank deficient, the error-correction form of the country-specific model (43) needs to be estimated subject to the reduced-rank restriction

$$H_{r_i}: \qquad rank(\boldsymbol{\Pi}_i) = r_i < k_i. \tag{46}$$

Under the assumption that $rank(\boldsymbol{\Pi}_i) = r_i$, we can write

$$\boldsymbol{\Pi}_i = \boldsymbol{\alpha}_i \boldsymbol{\beta}_i', \tag{47}$$

where $\boldsymbol{\alpha}_i$ is a $k_i \times r_i$ matrix of rank r_i and $\boldsymbol{\beta}_i$ is a $(k_i + k_i^* + s) \times r_i$ matrix of rank r_i.

For a given choice of $\boldsymbol{\beta}_i$, using (47) in (43), we have

$$\Delta \mathbf{x}_{it} = \mathbf{c}_{i0} - \boldsymbol{\alpha}_i \boldsymbol{\eta}_{it-1} + \boldsymbol{\Lambda}_{i0} \Delta \mathbf{x}_{it}^* + \boldsymbol{\Psi}_{i0} \Delta \mathbf{d}_t + \boldsymbol{\varepsilon}_{it}, \tag{48}$$

where

$$\boldsymbol{\eta}_{it} = \boldsymbol{\beta}_i' \mathbf{v}_{it} - (\boldsymbol{\beta}_i' \boldsymbol{\kappa}_i) t = \boldsymbol{\beta}_i' \mathbf{v}_{it} - \boldsymbol{\delta}_i t \tag{49}$$

is an $r_i \times 1$ vector of long-run or (detrended) cointegrating relations, also known as error-correction terms.

Identification and estimation of $\boldsymbol{\beta}_i$, and hence of other parameters, is carried out in two steps. First, the rank of $\boldsymbol{\Pi}_i$ is determined using, for example, the maximum eigenvalue or the trace statistics. Second, $\boldsymbol{\beta}_i$ is estimated by imposing suitable exact or possibly overidentifying restrictions on the elements of $\boldsymbol{\beta}_i$. Johansen's eigenvalue routine identifies $\boldsymbol{\beta}_i$ up to an $r_i \times r_i$ nonsingular matrix. To investigate the identification conditions in the present application, partition $\boldsymbol{\beta}_i$ as

$$\boldsymbol{\beta}_i = (\boldsymbol{\beta}_{ix}', \boldsymbol{\beta}_{ix^*}', \boldsymbol{\beta}_{id}')',$$

conformable to $\mathbf{v}_{it} = (\mathbf{x}_{it}', \mathbf{x}_{it}^{*\prime}, \mathbf{d}_t')'$. Then

$$\boldsymbol{\beta}_i' \mathbf{v}_{it} = \boldsymbol{\beta}_{ix}' \mathbf{x}_{it} + \boldsymbol{\beta}_{ix^*}' \mathbf{x}_{it}^* + \boldsymbol{\beta}_{id}' \mathbf{d}_t.$$

To identify $\boldsymbol{\beta}_i$, it is sufficient that $\boldsymbol{\beta}_{ix}$ (an $k_i \times r_i$ matrix)—namely, the part of $\boldsymbol{\beta}_i$ that corresponds to the endogenous variables, $\mathbf{x}_{it}$—be identified. Note that in general, it is also possible to identify $\boldsymbol{\beta}_i$ by placing restrictions on the other coefficients. For this purpose, we need a total of r_i^2 restrictions: r_i restrictions on each of the r_i columns of $\boldsymbol{\beta}_{ix}$. Notice that in the stationary case where $r_i = k_i$, identification of the long-run relations can be achieved by setting $\boldsymbol{\beta}_{ix}' = \mathbf{I}_{k_i}$. In cases where $r_i < k_i$, $\boldsymbol{\beta}_i$ can be exactly identified by setting $\boldsymbol{\beta}_{ix}' = (\mathbf{I}_{r_i} \vdots \mathbf{Q}_i)$, where $\mathbf{Q}_i$ is an $r_i \times (k_i - r_i)$ matrix of fixed coefficients to be estimated freely. Other types of identifying restrictions based on *a priori* economic theory can also be entertained. But all exactly identifying restrictions yield the same estimate of $\boldsymbol{\Pi}_i$, and hence for forecasting and impulse response analysis the results will be invariant to the choice of exact identifying restrictions (for a general discussion, see Pesaran and Shin 2002). In what follows we suggest using the exact identifying restrictions $\boldsymbol{\beta}_{ix}' = (\mathbf{I}_{r_i} \vdots \mathbf{Q}_i)$, which are relatively simple to implement.

For simulation of portfolio loss distributions (and for impulse response analysis), we also need to estimate the covariance matrix of $\boldsymbol{\varepsilon}_t$. Denote the reduced-rank regression estimates of $\boldsymbol{\varepsilon}_{it}$ by $\hat{\boldsymbol{\varepsilon}}_{it}$; we then have

$$\widehat{\text{cov}}(\boldsymbol{\varepsilon}_{it}, \boldsymbol{\varepsilon}_{jt}) = T^{-1} \sum_{t=1}^{T} \hat{\boldsymbol{\varepsilon}}_{it} \hat{\boldsymbol{\varepsilon}}_{jt}', \tag{50}$$

$$\widehat{\text{cov}}(\boldsymbol{\varepsilon}_t) = \begin{pmatrix} \widehat{\text{cov}}(\boldsymbol{\varepsilon}_{0t}, \boldsymbol{\varepsilon}_{0t}) & \widehat{\text{cov}}(\boldsymbol{\varepsilon}_{0t}, \boldsymbol{\varepsilon}_{1t}) & \cdots & \widehat{\text{cov}}(\boldsymbol{\varepsilon}_{0t}, \boldsymbol{\varepsilon}_{Nt}) \\ \widehat{\text{cov}}(\boldsymbol{\varepsilon}_{1t}, \boldsymbol{\varepsilon}_{0t}) & \widehat{\text{cov}}(\boldsymbol{\varepsilon}_{1t}, \boldsymbol{\varepsilon}_{1t}) & \cdots & \widehat{\text{cov}}(\boldsymbol{\varepsilon}_{1t}, \boldsymbol{\varepsilon}_{Nt}) \\ \vdots & \vdots & & \vdots \\ \widehat{\text{cov}}(\boldsymbol{\varepsilon}_{Nt}, \boldsymbol{\varepsilon}_{0t}) & \widehat{\text{cov}}(\boldsymbol{\varepsilon}_{Nt}, \boldsymbol{\varepsilon}_{1t}) & \cdots & \widehat{\text{cov}}(\boldsymbol{\varepsilon}_{Nt}, \boldsymbol{\varepsilon}_{Nt}) \end{pmatrix}, \tag{51}$$

$$\hat{\boldsymbol{\varepsilon}}_{it} = \mathbf{x}_{it} - \hat{\mathbf{a}}_{i0} - \hat{\mathbf{a}}_{i1} t - \hat{\boldsymbol{\Phi}}_i \mathbf{x}_{i,t-1} - \hat{\boldsymbol{\Lambda}}_{i0} \mathbf{x}_{it}^* - \hat{\boldsymbol{\Lambda}}_{i1} \mathbf{x}_{i,t-1}^* - \hat{\boldsymbol{\Psi}}_{i0} \mathbf{d}_t - \hat{\boldsymbol{\Psi}}_{i1} \mathbf{d}_{t-1}, \tag{52}$$

where $\hat{\mathbf{a}}_{i0}$, $\hat{\mathbf{a}}_{i1}$, $\hat{\boldsymbol{\Phi}}_i$, $\hat{\boldsymbol{\Lambda}}_{i0}$, $\hat{\boldsymbol{\Lambda}}_{i1}$, $\hat{\boldsymbol{\Psi}}_{i0}$, and $\hat{\boldsymbol{\Psi}}_{i1}$ are the country-specific reduced-rank estimates.

7.1 Testing Weak Exogeneity of $\mathbf{x}_{it}^*$

Given the partial nature of the foregoing analysis, it is important that the weak exogeneity of the foreign-specific variables be put to a test. Following Johansen (1992) and Boswijk (1992), we can check the weak exogeneity by testing the joint significance of the estimated error-correction terms, namely $\hat{\boldsymbol{\eta}}_{i,t-1} = \hat{\boldsymbol{\beta}}_i' \mathbf{v}_{i,t-1} - \hat{\boldsymbol{\delta}}_i (t-1)$ defined by (49), in the marginal models for the foreign-specific variables. For example, to test the weak exogeneity of the ℓth element of $\mathbf{x}_{it}^*$, the relevant marginal model is

$$\Delta x_{it,\ell}^* = c_{i\ell} + \boldsymbol{\alpha}_{i\ell}^* \hat{\boldsymbol{\eta}}_{i,t-1} + \sum_{j=1}^{p_{i\ell}} \boldsymbol{\delta}_{i\ell}' \Delta \mathbf{v}_{i,t-j} + \zeta_{it,\ell}, \tag{53}$$

where $\mathbf{v}_{it}$ is defined by (42). The lag order, $p_{i\ell}$, is set in the light of the empirical evidence and the available sample size. The weak exogeneity of $\Delta x_{it,\ell}^*$ can now be statistically evaluated by testing $\boldsymbol{\alpha}_{i\ell}^* = \mathbf{0}$, using standard F tests, in a procedure similar to one advocated by Harbo et al. (1998, p. 395).

Finally, it is worth noting that even if the weak exogeneity assumption is rejected, one could still obtain consistent estimates of the parameters of the GVAR model in two steps. First, the country-specific models can be estimated treating all of the domestic- and foreign-specific variables (as well as the common global variables if deemed necessary) as endogenous. These parameter estimates can then be used to obtain the parameters of the conditional models, $\mathbf{x}_i | \mathbf{x}_i^*$, separately for $i = 0, 1, \ldots, N$, which can then be used to estimate the parameters of the full GVAR model. This approach is more data intensive, however, and will not be efficient if the weak exogeneity assumption is met in the case of one or more of the variables. A mixed estimation strategy, treating some but not all of the foreign-specific variables as weakly exogenous, is also clearly feasible.

8. CROSS–COUNTRY AGGREGATION IN GVAR MODELING

One of the strengths of the GVAR modeling approach is its flexibility to take into account the various interlinkages in the global economy in the context of a truly multicountry setting. But this approach can be demanding in terms of data management, computations, and data analysis when a large number of countries (say 100 or more) are included in the model. One possible way of making the analysis more manageable would be to apply the approach to a few key countries (say, the G7) individually, and then aggregate the remaining countries into 5–10 blocks or regions. This section considers how regional models can be constructed from the underlying country-specific models, which, to be sure, is a matter of convenience and not a logical requirement of the model.

Consider a given region i (e.g., Southeast Asia, North Africa, the Middle East) composed of N_i countries. Denote the vector of country-specific variables in region i by $\mathbf{x}_{i\ell t}$, and that of the associated foreign variables by $\mathbf{x}^*_{i\ell t}$, where $i = 0, 1, 2, \ldots, R$ and $\ell = 1, 2, \ldots, N_i$. We continue to assume that the reference country (or region) is denoted by 0. The country-specific model for country ℓ in region i is given by

$$\mathbf{x}_{i\ell t} = \mathbf{a}_{i\ell 0} + \mathbf{a}_{i\ell 1}t + \boldsymbol{\Phi}_{i\ell}\mathbf{x}_{i\ell,t-1} + \boldsymbol{\Lambda}_{i\ell 0}\mathbf{x}^*_{i\ell t} + \boldsymbol{\Lambda}_{i\ell 1}\mathbf{x}^*_{i\ell,t-1} + \boldsymbol{\Psi}_{i\ell 0}\mathbf{d}_t + \boldsymbol{\Psi}_{i\ell 1}\mathbf{d}_{t-1} + \boldsymbol{\varepsilon}_{i\ell t}, \quad (54)$$

which is an adaptation of (1). The problem of aggregating the N_i countries within region i centers on the heterogeneity of the coefficient matrices $\boldsymbol{\Phi}_{i\ell}$, $\boldsymbol{\Lambda}_{i\ell 0}$, and $\boldsymbol{\Lambda}_{i\ell 1}$ associated with the country-specific variables. The cross-country heterogeneity of the remaining parameters does not pose any special problem. There will always be an aggregation problem as long as $\boldsymbol{\Phi}_{i\ell}$, $\boldsymbol{\Lambda}_{i\ell 0}$, and $\boldsymbol{\Lambda}_{i\ell 1}$ differ across the countries in the region. But in practice it is possible to reduce the size of the aggregation error by using a weighted average of the variables $\mathbf{x}_{i\ell t}$ (and hence of $\mathbf{x}^*_{i\ell t}$), with the weights reflecting the relative importance of the countries in the region. Let $w^0_{i\ell}$ be the weight of country ℓ in the region i. Clearly, $\sum_{\ell=1}^{N_i} w^0_{i\ell} = 1$. Then, aggregating the countries in the region using these weights, we have

$$\mathbf{x}_{it} = \mathbf{a}_{i0} + \mathbf{a}_{i1}t + \sum_{\ell=1}^{N_i} w^0_{i\ell}\boldsymbol{\Phi}_{i\ell}\mathbf{x}_{i\ell,t-1} + \sum_{\ell=1}^{N_i} w^0_{i\ell}\boldsymbol{\Lambda}_{i\ell 0}\mathbf{x}^*_{i\ell t} + \sum_{\ell=1}^{N_i} w^0_{i\ell}\boldsymbol{\Lambda}_{i\ell 1}\mathbf{x}^*_{i\ell,t-1} + \boldsymbol{\Psi}_{i0}\mathbf{d}_t + \boldsymbol{\Psi}_{i1}\mathbf{d}_{t-1} + \boldsymbol{\varepsilon}_{it}, \quad (55)$$

where

$$\mathbf{x}_{it} = \sum_{\ell=1}^{N_i} w^0_{i\ell}\mathbf{x}_{i\ell t}, \qquad \mathbf{a}_{i0} = \sum_{\ell=1}^{N_i} w^0_{i\ell}\mathbf{a}_{i\ell 0}, \qquad \mathbf{a}_{i1} = \sum_{\ell=1}^{N_i} w^0_{i\ell}\mathbf{a}_{i\ell 1} \quad (56)$$

and

$$\boldsymbol{\Psi}_{i0} = \sum_{\ell=1}^{N_i} w^0_{i\ell}\boldsymbol{\Psi}_{i\ell 0}, \qquad \boldsymbol{\Psi}_{i1} = \sum_{\ell=1}^{N_i} w^0_{i\ell}\boldsymbol{\Psi}_{i\ell 1}, \qquad \boldsymbol{\varepsilon}_{it} = \sum_{\ell=1}^{N_i} w^0_{i\ell}\boldsymbol{\varepsilon}_{i\ell t}. \quad (57)$$

Using (55), a regional model as specified in (1) can be obtained. In terms of the foregoing notations, we have

$$\mathbf{x}_{it} = \mathbf{a}_{i0} + \mathbf{a}_{i1}t + \boldsymbol{\Phi}_i\mathbf{x}_{i,t-1} + \boldsymbol{\Lambda}_{i0}\mathbf{x}^*_{it} + \boldsymbol{\Lambda}_{i1}\mathbf{x}^*_{i,t-1} + \boldsymbol{\Psi}_{i0}\mathbf{d}_t + \boldsymbol{\Psi}_{i1}\mathbf{d}_{t-1} + \boldsymbol{\xi}_{it}, \quad (58)$$

where $\boldsymbol{\xi}_{it} = \boldsymbol{\varepsilon}_{it} + v_{it}$ is now composed of the equation errors, $\boldsymbol{\varepsilon}_{it}$, and the aggregation error is defined by

$$v_{it} = \sum_{\ell=1}^{N_i} w^0_{i\ell}(\boldsymbol{\Phi}_{i\ell} - \boldsymbol{\Phi}_i)\mathbf{x}_{i\ell,t-1} + \sum_{\ell=1}^{N_i} w^0_{i\ell}(\boldsymbol{\Lambda}_{i\ell 0} - \boldsymbol{\Lambda}_{i0})\mathbf{x}^*_{i\ell t} + \sum_{\ell=1}^{N_i} w^0_{i\ell}(\boldsymbol{\Lambda}_{i\ell 1} - \boldsymbol{\Lambda}_{i1})\mathbf{x}^*_{i\ell,t-1}. \quad (59)$$

The region-specific foreign variables, $\mathbf{x}^*_{it}$, can be constructed using either regional trade weights or country-specific trade weights as in (4). In the case of the latter, y^*_{it}, for example, is defined as

$$y^*_{it} = \sum_{\ell=1}^{N_i} w^0_{i\ell} y^*_{i\ell t}, \qquad i = 0, 1, 2, \ldots, R, \quad (60)$$

where

$$y^*_{i\ell t} = \sum_{j=0}^{R}\sum_{k=1}^{N_j} w^y_{i\ell,jk} y_{jkt}, \qquad \ell = 1, 2, \ldots, N_i, i = 0, 1, 2, \ldots, R, \quad (61)$$

and $w^y_{i\ell,jk}$ is the share of country k in region j in the total trade of country ℓ in region i,

$$N = \sum_{i=0}^{R} N_i. \quad (62)$$

The importance of the aggregation error depends on the extent and nature of the differences in the coefficient matrices $\boldsymbol{\Phi}_{i\ell}$, $\boldsymbol{\Lambda}_{i\ell 0}$, and $\boldsymbol{\Lambda}_{i\ell 1}$ across the different countries in the region. The aggregation error can be minimized by choosing regions with similar economies (as much as possible) and by a sensible choice of the weights, $w^0_{i\ell}$. The importance of countries in a region is best measured by their output levels, and for comparability, it is important that they be measured in purchasing power parity (PPP) dollars. The weights $w^0_{i\ell}$ can be computed using PPP-adjusted GDP series for a given year or based on averages computed over several years. It may also be desirable to update the weights on a rolling basis, say, by using 5-year lagged moving averages.

In view of the foregoing analysis, the regional variables can be constructed from country-specific variables using the following (logarithmic) weighted averages

$$y_{it} = \sum_{\ell=1}^{N_i} w^0_{i\ell} y_{i\ell t}, \qquad p_{it} = \sum_{\ell=1}^{N_i} w^0_{i\ell} p_{i\ell t}, \qquad q_{it} = \sum_{\ell=1}^{N_i} w^0_{i\ell} q_{i\ell t} \quad (63)$$

and

$$e_{it} = \sum_{\ell=1}^{N_i} w_{i\ell}^0 e_{i\ell t}, \qquad \rho_{it} = \sum_{\ell=1}^{N_i} w_{i\ell}^0 \rho_{i\ell t}, \\ m_{it} = \sum_{\ell=1}^{N_i} w_{i\ell}^0 m_{i\ell t}. \tag{64}$$

Notice that in constructing the regional variables $y_{it}, p_{it}, e_{it}, \ldots$ from the country-specific variables $y_{i\ell t}, p_{i\ell t}, e_{i\ell t}, \ldots$, one simply needs to use country-specific variables measured in their domestic currencies, bearing in mind that $e_{i\ell t}$ stands for the exchange rate of country ℓ in region i, measured in U.S. dollar.

9. AN EMPIRICAL APPLICATION

9.1 Countries and Regions

In this section we illustrate our approach by estimating a global quarterly model over the period 1979Q1–1999Q1 comprising the U.S., Germany, France, Italy, the U.K., Japan, China, and 18 other countries aggregated into four regions: Western Europe, Southeast Asia, Middle East, and Latin America. The details of these 11 country/region classifications are given in Table 1.

The output from these countries comprises around 82% of world GDP in 1999. They were chosen largely because the major banks in G-7 countries have most of their exposure in this set of countries. Noticeably absent are Scandinavian countries, Africa, and Australia–New Zealand. Future extensions of the model will look to incorporate countries from these regions. Time series data on regions such as Latin America or Southeast Asia were constructed from each country in the region weighted by the GDP share. For this, we used PPP-weighted GDP figures, which are thought to be more reliable than weights based on U.S. dollar GDPs. Information on data sources and the construction of regional data series are provided in Appendix A. Also see Section 8 for details of regional aggregation.

For modeling purposes, we distinguish between the regions with developed capital markets (U.S., Germany, Japan, Western European countries, Southeast Asia, and Latin America) and the rest (China and Middle East), which, over our sample period, did not have fully functioning capital markets. Finally, as noted earlier, we use the U.S. dollar as the numeraire exchange rate and determine its value in terms of the other currencies outside the U.S. model.

Table 1. Countries/Regions in the GVAR Model

U.S.	**Germany**	**Japan**
Western Europe • Spain • Belgium • Netherlands • Switzerland	**Southeast Asia** • Korea • Thailand • Indonesia • Malaysia • Philippines • Singapore	**Latin America** • Argentina • Brazil • Chile • Peru • Mexico
Middle East • Kuwait • Saudi Arabia • Turkey	**China** **U.K.**	**France** **Italy**

9.2 The Trade Weights

The first step in the GVAR modeling exercise is to construct the foreign country/region-specific ("starred") variables from the domestic variables using the relations (4). For the weights, we decided to rely exclusively on trade weights based on the United Nations Direction of Trade Statistics. Information on capital flows were not of sufficiently high quality and tended to be rather volatile. The 11×11 matrix of the trade weights computed as shares of exports and imports over the 1996–1998 period is presented in Table 2.

The trade shares of each country/region is displayed in columns. This matrix plays a key role in linking up the models of the different regions together and shows the degree to which one country/region depends on the remaining countries. For example, not surprisingly the trade weights show that Latin America is much more integrated with the U.S. economy than the rest of the regions, whereas the Middle East is more integrated with the economies of Western Europe and Germany and the bulk of China's trade is with the U.S., Germany, Japan, and Southeast Asia.

9.3 Integration Properties of the Series

The second step in the modeling process is to select appropriate transformations of the domestic and foreign variables for inclusion in the country/region-specific cointegrating VAR models. The reduced-rank regression techniques reviewed in Section 7 are based on the assumption that the underlying endogenous and exogenous variables to be included in the country/region-specific models are approximately integrated of order unity. To ascertain the order of integration of the variables in the country/region-specific models in Table 3, we present augmented Dickey–Fuller (ADF) statistics for the levels, first differences, and the second differences of the domestic and country/region-specific foreign variables. A number of modifications of the ADF test have also been proposed in the literature by, for example, Pantula, Gonzalez-Farias, and Fuller (1994), Leybourne (1995), and Elliot, Rothenberg, and Stock (1996), which have been shown to have better small-sample power characteristics. To check the robustness of our conclusions to the choice of the test statistics, we also computed Elliott et al.'s (1996) ADF–GLS statistics for all of the series reported in Table 3. The test results based on ADF and ADF–GLS statistics differ in only a few cases, and there seems to be no obvious patterns to these differences. Therefore, in what follows we focus on the ADF test results. (The ADF–GLS statistics are available from the authors on request.)

To ensure comparability, all of these statistics are computed over the same period, 1980Q2–1999Q1, starting with an underlying univariate AR process of order 5, with a linear trend in the case of the levels (except for the interest rates) and an intercept term only in the case of the first and second differences. The orders of the ADF test statistics reported in Table 3 are selected according to the Akaike information criterion (AIC).

Generally speaking, the results of these unit root tests are in line with what is known in the literature. Interest rates (domestic and foreign) and real equity prices (domestic and foreign) are unambiguously $I(1)$ across all countries/regions. The same also applies to exchange rates, with the notable exception

Table 2. Trade Weights (ω_{ij}) Based on Direction of Trade Statistics

Country/ region	*U.S.*	*U.K.*	*Germany*	*France*	*Italy*	*Western Europe*	*Middle East*	*China*	*Southeast Asia*	*Japan*	*Latin America*
U.S.	0	.1889	.1233	.0995	.0967	.1025	.1970	.2371	.3262	.3528	.6799
U.K.	.0791	0	.1164	.1211	.1020	.1395	.0956	.0280	.0518	.0384	.0225
Germany	.0809	.1825	0	.2121	.2371	.3007	.1196	.0613	.0636	.0601	.0494
France	.0434	.1260	.1408	0	.1729	.1923	.0602	.0270	.0339	.0250	.0227
Italy	.0304	.0765	.1135	.1247	0	.1060	.0705	.0227	.0206	.0163	.0332
Western Europe	.0731	.2376	.3267	.3147	.2405	0	.1048	.0595	.0636	.0477	.0658
Middle East	.0295	.0342	.0273	.0207	.0336	.0220	0	.0122	.0452	.0391	.0075
China	.0509	.0144	.0201	.0133	.0155	.0179	.0175	0	.0869	.1104	.0203
Southeast Asia	.1825	.0692	.0542	.0435	.0367	.0499	.1688	.2264	0	.2768	.0419
Japan	.2019	.0525	.0524	.0329	.0296	.0382	.1493	.2941	.2831	0	.0571
Latin America	.2282	.0180	.0253	.0175	.0354	.0309	.0168	.0317	.0251	.0335	0

NOTE: Trade weights are computed as shares of exports and imports, displayed in columns by region (such that a column, but not a row, sum to 1).

Source: *International Trade Statistics Yearbook*, 1996–1998, New York: United Nations.

Table 3. ADF Unit Root Test Statistics (based on AIC order selection)

Domestic variables	*U.S.*	*U.K.*	*Germany*	*France*	*Italy*	*Western Europe*	*Middle East*	*China*	*Southeast Asia*	*Japan*	*Latin America*
y	−2.54	−2.94	−2.60	−2.78	−1.27	−3.39	−3.10	−3.79	−2.19	−.29	−3.29
Δy	−5.85	−3.01	−9.07	−3.87	−6.94	−3.91	−5.39	−2.41	−4.31	−2.12	−3.50
$\Delta^2 y$	−9.34	−10.42	−10.01	−16.60	−7.53	−9.80	−8.28	−14.26	−2.82	−10.30	−7.56
p	−1.91	−1.42	−2.89	−4.30	−2.25	−2.16	−1.33	−2.70	−2.57	−2.49	−1.51
Δp	−3.93	−4.17	−1.82	−1.99	−2.63	−1.60	−4.04	−2.04	−3.41	−3.32	−2.41
$\Delta^2 p$	−12.32	−8.78	−8.95	−8.29	−12.09	−9.86	−9.70	−8.56	−6.71	−7.99	−10.41
e		−2.61	−2.42	−2.84	−2.69	−2.53	−.94	−1.56	−1.87	−2.01	−1.82
Δe		−6.96	−6.31	−5.92	−6.07	−6.17	−8.47	−8.16	−5.93	−6.73	−2.33
$\Delta^2 e$		−7.95	−7.84	−7.77	−7.89	−7.48	−11.73	−8.53	−7.58	−10.98	−11.21
$e - p$		−2.44	−2.26	−2.16	−1.73	−1.97	−2.07	−1.31	−2.14	−1.80	−1.83
$\Delta(e - p)$		−6.91	−6.60	−6.45	−6.62	−6.46	−7.31	−7.76	−6.07	−6.79	−3.68
$\Delta^2(e - p)$		−7.81	−7.99	−7.88	−8.08	−7.53	−9.61	−8.47	−8.00	−11.02	−10.07
ρ	−2.10	−1.73	−1.95	−.76	.23	−.50	−2.32	−1.49	−3.77	−1.48	−3.07
$\Delta \rho$	−7.75	−9.47	−4.80	−7.65	−8.24	−8.91	−9.08	−6.45	−4.78	−5.92	−8.50
$\Delta^2 \rho$	−8.40	−10.90	−7.39	−8.85	−8.70	−9.29	−10.45	−7.98	−8.94	−9.00	−9.13
m	−1.81	−.96	−2.91	−1.29	−3.08	−2.15	−1.53	−3.30	−1.96	−1.18	−2.33
Δm	−4.25	−8.87	−7.67	−2.44	−2.26	−7.38	−3.48	−2.42	−8.78	−3.04	−3.38
$\Delta^2 m$	−7.79	−8.73	−10.51	−9.67	−12.87	−9.09	−8.33	−17.23	−8.21	−9.71	−9.81
q	−1.77	−1.91	−2.74	−2.45	−2.74	−1.81			−1.84	−1.01	−4.01
Δq	−7.27	−7.98	−7.14	−6.43	−4.64	−3.95			−3.12	−3.58	−8.21
$\Delta^2 q$	−7.75	−7.92	−11.68	−10.50	−9.65	−12.70			−10.87	−10.63	−10.64
y^*		−2.65	−2.62	−2.45	−3.00	−2.61	−1.80	−1.28	−1.81	−2.72	−2.76
Δy^*		−5.70	−4.55	−5.21	−3.93	−6.56	−4.89	−2.62	−6.64	−4.09	−5.75
$\Delta^2 y^*$		−9.10	−8.32	−9.26	−9.62	−9.95	−9.12	−9.82	−7.88	−8.37	−9.55
p^*		−.24	.47	.05	−.03	.14	−.55	−.65	−1.39	−1.43	−1.75
Δp^*		−2.52	−2.32	−2.49	−2.49	−2.12	−3.00	−3.49	−2.24	−3.50	−4.24
$\Delta^2 p^*$		−9.87	−8.82	−10.09	−8.00	−8.59	−8.72	−8.04	−9.35	−10.85	−12.09
e^*	−1.22	−2.65	−2.16	−3.02	−2.77	−2.33	−1.83	−.97	−2.18	−2.01	−1.82
Δe^*	−4.89	−6.51	−5.81	−5.78	−6.11	−5.34	−8.19	−8.08	−5.44	−6.86	−2.30
$\Delta^2 e^*$	−7.77	−8.02	−10.01	−10.31	−9.96	−8.95	−8.74	−8.21	−7.54	−11.21	−11.36
$e^* - p^*$	−2.10	−3.75	.29	−2.13	−1.54	−.61	−1.79	−1.44	−2.00	−1.77	−1.86
$\Delta(e^* - p^*)$	−6.73	−6.43	−5.39	−5.84	−6.04	−4.74	−7.73	−8.10	−5.10	−3.84	−2.32
$\Delta^2(e^* - p^*)$	−9.17	−7.93	−10.06	−10.27	−9.73	−8.52	−8.58	−8.16	−7.59	−11.53	−11.34
ρ^*		−1.39	−1.45	−1.04	−1.99	−1.78	−1.36	−2.22	−1.01	−3.04	−1.80
$\Delta \rho^*$		−7.87	−8.17	−7.56	−8.15	−8.15	−7.49	−8.42	−8.49	−8.35	−7.33
$\Delta^2 \rho^*$		−9.41	−9.37	−9.14	−9.20	−9.26	−8.97	−9.43	−9.47	−9.48	−8.19
m^*		−2.01	−2.62	−2.36	−2.61	−2.24	−2.10	−1.59	−2.56	−2.07	−2.03
Δm^*		−6.25	−2.37	−2.48	−2.69	−3.02	−2.73	−3.24	−2.25	−3.12	−5.72
$\Delta^2 m^*$		−8.65	−12.29	−11.33	−10.90	−10.33	−9.87	−9.65	−12.38	−11.08	−9.95
q^*		−2.25	−2.18	−1.77	−2.35	−2.13	−2.11	−1.36	−1.86	−2.23	−2.09
Δq^*		−6.41	−6.79	−6.88	−4.04	−6.57	−4.54	−4.44	−6.25	−4.71	−6.82
$\Delta^2 q^*$		−10.58	−10.81	−10.57	−10.78	−10.52	−10.48	−10.50	−10.06	−11.02	−9.63

NOTE: The ADF statistics are based on univariate AR(p) models in the level of the variables with $p \leq 5$, and the statistics for the level, first differences, and second differences of the variables are all computed on the basis of the same sample period, namely 1980Q2–1999Q1. The ADF statistics for all the level variables are based on regressions including a linear trend, except for the interest variables. The 95% critical value of the ADF statistics for regressions with trend is −3.47, and for regressions without trend −2.90.

of Latin America. In the case of Latin America, the hypothesis that exchange rate is $I(2)$ cannot be rejected (see the last column of Table 3). There are two possible ways of dealing with this problem. We could decide to model Δe instead of e, but this will most likely involve overdifferencing and efficiency loss in the case of the remaining regional models. Another, arguably more attractive, alternative would be to include the real exchange rate $(e - p)$ in the regional models. The hypothesis that $e - p$ is $I(1)$ now prevails across all countries, and the hypothesis that $e^* - p^*$ is $I(1)$ is not supported only in the case of U.K. and Latin America. The integration property of $e^* - p^*$ is only relevant for the U.S. model as it is not included elsewhere. In the case of the U.S., e^* is defined by

$$e^*_{US} = \sum_{j=1}^{10} w_{US,j} e_j, \tag{65}$$

is an $I(2)$ variable (see the last column of Table 3). There are two possible ways of dealing with this problem. We could decide to model Δe instead of e, but this will most likely involve overdifferencing and efficiency loss in the case of the seven remaining regional models. Another, arguably more attractive, alternative would be to include the real exchange rate $(e - p)$ in the regional models. The hypothesis that $e - p$ is $I(1)$ now prevails across all countries, and the hypothesis that $e^* - p^*$ is $I(1)$ is not supported only in the cases of U.K. and Latin America.

As far as the order of integration of the remaining three variables are concerned, the evidence is less clear cut, which is due partly to uneven data quality across the countries and the relatively short sample period under consideration. Using the 95% significance level, a unit root in real output is not rejected in any of the 11 regions. However, in the case of Japan and China, the ADF statistics seem to suggest that real output could be $I(2)$! This is clearly implausible and again could be due to poor data quality in the case of China. The result for Japan is difficult to explain, however, although Japan's national income statistics are not consider particularly reliable. A similar argument also applies to foreign output variables, y^*, and real money balances, m and m^*. Overall, however, it seems appropriate for our purposes to treat all of these variables approximately as $I(1)$. Finally, for the price variables, the test results suggest that the general price level, p, is $I(1)$ in six regions and $I(2)$ in the remaining five regions. A similar outcome prevails with respect to p^*, which is $I(2)$ for six countries and $I(1)$ for the remaining four. (Recall that p^* is not included in the U.S. model.) Because overdifferencing is likely to be less serious for the empirical analysis than wrongly including an $I(2)$ instead of an $I(1)$ variable, we use inflation rates, Δp and Δp^*, that are at most $I(1)$ instead of the price levels.

9.4 Country/Region-Specific Models

In view of these results, we selected the endogenous variables of the U.S. model as real output (y_{US}), inflation rate (Δp_{US}), interest rate variable (ρ_{US}), real money balances (m_{US}), and real equity prices (q_{US}), all measured in logarithms as defined in (3). Within the GVAR framework, the value of the U.S. dollar is determined outside the U.S. model, and the U.S.-specific real exchange rate variable, $e^*_{\text{US}} - p^*_{\text{US}}$, is then included as an $I(1)$ weakly exogenous variable in the U.S. model. The weights $w_{\text{US},j}$, $j = 1, 2, \dots, 10$, are given in the first column of Table 2. Given the size of the U.S. economy and its importance for global economic interactions, no other foreign-specific exogenous variable was considered for inclusion in the U.S. model. But to control for important global political events, the logarithm of oil prices (p^o) were included as an exogenous $I(1)$ variable in *all* of the country/region specific models. The ADF statistics computed over the period 1980Q2–1999Q1 for the level and first differences of oil prices were -2.27 and -4.74, thus providing empirical support for treating oil prices as an $I(1)$ variable. The GVAR model can be made into a closed system by including oil prices in the U.S. model as an endogenous $I(1)$ variable, while retaining it as an exogenous $I(1)$ variable in the remaining regional models. Such a specification would be particularly convenient for forecasting purposes and allows for possible long-term feedbacks from the U.S. macro variables into the determination of oil prices.

In the case of the U.K., Germany, France, Italy, the rest of Western Europe, Japan, Southeast Asia, and Latin America with advanced capital markets, we chose $(y_j, \Delta p_j, \rho_j, e_j - p_j, m_j, q_j)$ and $(y^*_j, \Delta p^*_j, \rho^*_j, m^*_j, q^*_j, p^o)$ as their endogenous and exogenous variables. Notice that e^*_j is excluded from the set of exogenous variables on the grounds of its close relationship to e_j. For the remaining regions (Middle East and China), the sets of included endogenous and exogenous variables were $(y_j, \Delta p_j, \rho_j, e_j - p_j, m_j)$ and $(y^*_j, \Delta p^*_j, \rho^*_j, m^*_j, q^*_j, p^o)$.

The next step in the analysis is to estimate region-specific cointegrating VAR models and identify the rank of their cointegrating space. We took the order of the underlying VAR models to be 1. This choice was dictated to us by the small number of time series observations that were available to us relative to the number of unknown parameters in each of the regional models. The "trace" and "maximum eigenvalue" test statistics for each of the 11 regions together with the associated 90% and 95% critical values are summarized in Tables 4–6. These statistics are computed using VAR(1) specifications with restricted trend coefficients. This is model IV of Pesaran et al. (2000). Computations were carried out using Microfit 4.1 (see Pesaran and Pesaran 1997).

It is known that both of these statistics tend to overreject in small samples, with the extent of overrejection being much more serious for the maximum eigenvalue as compared to the trace test. Using Monte Carlo experiments, it has also been shown that the maximum eigenvalue test is generally less robust to departures from normal errors than the trace test (see, e.g., Cheung and Lai 1993). The latter point is particularly relevant to our applications, because they contain equity prices, exchange rates, and interest rates, all of which exhibit significant degrees of departures from normality. We therefore base our inference on the trace statistics. Accordingly, we found five cointegrating relations for the U.K.; four for Germany and Japan; three for Italy, Western Europe, Southeast Asia, Latin America, Middle East, and China; and two for France and the U.S. The result for the U.K. is in line with the full system estimates reported by Garratt et al. (2003a) for the U.K. over the period 1965Q1–1999Q4. For France, the trace test when applied at 90% is very marginal and in view of the three or more

Table 4. Cointegration Rank Statistics for Regions With Capital Markets

H_0	H_1	U.K.	Germany	France	Italy	Western Europe	Southeast Asia	Japan	Latin America	Critical value 95%	Critical value 90%
Maximum eigenvalue statistics											
$r=0$	r = 1	130.16	114.29	75.90	72.67	90.57	145.48	94.82	144.99	61.74	58.48
$r<1$	r = 2	61.42	75.00	62.48	62.02	75.80	95.68	78.97	65.95	55.40	52.18
$r\leq 2$	r = 3	55.66	61.14	42.57	55.89	55.13	43.59	53.59	44.72	49.16	46.08
$r\leq 3$	r = 4	47.90	55.17	32.56	25.80	38.44	35.38	41.51	30.49	42.91	40.06
$r\leq 4$	r = 5	39.22	33.45	21.30	18.22	21.02	22.65	26.93	27.13	36.02	33.10
$r\leq 5$	r = 6	25.42	15.68	13.88	16.58	10.97	21.68	17.86	16.67	28.57	25.55
Trace statistics											
$r=0$	r > 1	359.78	354.73	248.70	251.17	291.92	364.47	313.68	329.94	191.45	184.80
$r<1$	r ≥ 2	229.62	240.44	172.79	178.50	201.35	218.99	218.87	184.95	152.15	145.67
$r\leq 2$	r ≥ 3	168.20	165.44	110.32	116.48	125.55	123.31	139.90	119.00	115.43	110.31
$r\leq 3$	r ≥ 4	112.54	104.30	67.75	60.59	70.42	79.72	86.31	74.29	83.43	78.85
$r\leq 4$	r ≥ 5	64.64	49.13	35.19	34.80	31.98	44.33	44.79	43.80	54.21	50.39
$r\leq 5$	r ≥ 6	25.42	15.68	13.88	16.58	10.97	21.68	17.86	16.67	28.57	25.55

NOTE: The model contains unrestricted intercepts and restricted trend coefficients with $I(1)$ endogenous variables y, Δp, q, $e-p$, ρ, and m, and $I(1)$ exogenous variables y^*, Δp^*, q^*, ρ^*, m^*, p^o.

cointegrating relations found for other Western European countries could be an underestimate. So in what follows we also assume that there are three cointegrating relations in the model for France. For the U.S., the test results seem quite conclusive, and given the particular nature of the U.S. model, we did not see any ground for doubting the two cointegrating relations suggested by the tests. The cointegrating relations can be interpreted as long-run relations, either among the domestic variables and/or between the domestic and foreign variables. The long-run money demand equation (that relates m_{it} to ρ_{it} and y_{it}) is an example of the former, whereas the uncovered interest parity (that relates ρ_{it} to ρ^*_{it}) provides an example of the latter. These theoretical long-run relations suggest further (overidentifying) restrictions on the cointegrating relations that can be imposed and tested as done by, for example, Garratt et al. (2003a). However, this will require detailed long-run structural analysis of the individual regions, which is beyond the scope of the present application.

9.5 Testing Weak Exogeneity of the Country-Specific Foreign Variables

One of the key assumptions underlying our estimation approach is the weak exogeneity of the country-specific foreign variables. But, as noted earlier, this assumption can be tested by running first-difference regressions of the foreign variables and testing the significance of the country-specific error-correction terms in these regressions [see (53)]. For example, to the test the weak exogeneity of, for instance, foreign output in the U.K. model, y^*_{UK}, we need to test the joint hypothesis that

$$\delta_{\text{UK},j}=0, \qquad j=1,2,\ldots,5$$

in the regression

$$\Delta y^*_{\text{UK},t}=a_{\text{UK}}+\sum_{j=1}^{5}\delta_{\text{UK},j}ECM^{(j)}_{\text{UK},t-1}+\boldsymbol{\phi}'_{\text{UK}}\Delta\mathbf{z}_{\text{UK},t-1}+\phi_{\text{UK},o}\Delta p^o_{t-1}+\zeta_{\text{UK},t},$$

where $ECM^{(j)}_{\text{UK},t-1}$, $j=1,2,\ldots,5$, are the estimated error-correction terms associated with the five cointegrating relations found in the U.K. model, $\Delta\mathbf{z}_{\text{UK},t-1}=(\Delta\mathbf{x}'_{\text{UK},t-1}, \Delta\mathbf{x}'^*_{\text{UK},t-1}, \Delta e^*_{\text{UK},t-1}-\Delta p^*_{\text{UK},t-1})'$. The F-statistics for testing the weak exogeneity of all of the country-specific foreign variables and the oil price variable are summarized in Table 7. Of the 62 weak exogeneity tests carried out, only 2 were found to be statistically significant at the 5% level and none at 3% or less. The two rejections of weak exogeneity assumption relate to foreign output

Table 5. Cointegration Rank Statistics for Regions Without Capital Markets

H_0	H_1	Middle East	China	Critical value 95%	Critical value 90%
Maximum eigenvalue statistics					
$r=0$	r = 1	72.52	51.14	55.40	52.18
$r<1$	r = 2	62.41	47.52	49.16	46.08
$r\leq 2$	r = 3	51.86	38.20	42.91	40.06
$r\leq 3$	r = 4	32.62	29.49	36.02	33.10
$r\leq 4$	r = 5	18.75	18.08	28.57	25.55
Trace statistics					
$r=0$	r > 1	238.16	184.43	152.15	145.67
$r<1$	r ≥ 2	165.64	133.29	115.43	110.31
$r\leq 2$	r ≥ 3	103.23	85.77	83.43	78.85
$r\leq 3$	r ≥ 4	51.37	47.57	54.21	50.39
$r\leq 4$	r ≥ 5	18.75	18.08	28.57	25.55

NOTE: The model contains unrestricted intercepts and restricted trend coefficients with $I(1)$ endogenous variables y, Δp, $e-p$, ρ, and m and $I(1)$ exogenous variables y^*, Δp^*, q^*, ρ^*, m^*, and p^o.

Table 6. Cointegration Rank Statistics for the U.S. Model

H_0	H_1	U.S.	Critical value 95%	Critical value 90%
Maximum eigenvalue statistics				
$r=0$	$r=1$	85.91	43.72	40.94
$r<1$	$r=2$	51.99	37.85	35.04
$r\leq 2$	$r=3$	25.13	31.68	29.00
$r\leq 3$	$r=4$	15.18	24.88	22.53
$r\leq 4$	$r=5$	5.24	18.08	15.82
Trace statistics				
$r=0$	$r>1$	183.45	108.90	103.71
$r<1$	$r\geq 2$	97.54	81.20	76.68
$r\leq 2$	$r\geq 3$	45.55	56.43	52.71
$r\leq 3$	$r\geq 4$	20.42	35.37	32.51
$r\leq 4$	$r\geq 5$	5.24	18.08	15.82

NOTE: The model contains unrestricted intercepts and restricted trend coefficients with $I(1)$ endogenous variables y, Δp, q, ρ, and m and $I(1)$ exogenous variables e^*-p^* and p^o.

Table 7. F Statistics for Testing the Weak Exogeneity of the Country-Specific Foreign Variables and Oil Prices

	Foreign variables						
Country/region	y^*	Δp^*	ρ^*	m^*	q^*	p^o	$e^* - p^*$
U.S.							
$F(2, 68)$						3.51*	.67
						[.035]	[.513]
U.K.							
$F(5, 59)$	1.19	.70	.36	1.26	.48	2.32	
	[.326]	[.623]	[.872]	[.295]	[.788]	[.055]	
Germany							
$F(4, 60)$	1.03	1.14	.58	1.49	1.21	.67	
	[.398]	[.345]	[.676]	[.217]	[.317]	[.618]	
France							
$F(3, 61)$	2.82*	1.26	1.20	.68	.81	1.31	
	[.046]	[.297]	[.317]	[.565]	[.492]	[.279]	
Italy							
$F(3, 61)$	1.18	.36	.87	1.20	1.35	.48	
	[.324]	[.781]	[.462]	[.317]	[.266]	[.697]	
Western Europe							
$F(3, 61)$	.71	.08	.64	.37	.59	.03	
	[.551]	[.970]	[.590]	[.771]	[.624]	[.994]	
Middle East							
$F(3, 62)$	.12	.77	.32	.23	.24	.17	
	[.950]	[.514]	[.814]	[.875]	[.869]	[.914]	
China							
$F(3, 62)$	2.50	1.00	.20	.61	.11	.26	
	[.068]	[.399]	[.896]	[.611]	[.954]	[.850]	
Southeast Asia							
$F(3, 61)$	.29	.94	.28	.34	.62	2.03	
	[.841]	[.428]	[.842]	[.799]	[.605]	[.118]	
Japan							
$F(4, 60)$	.15	.31	1.20	.34	.71	1.10	
	[.962]	[.869]	[.322]	[.851]	[.588]	[.363]	
Latin America							
$F(3, 61)$	.94	.68	1.22	.31	2.47	.08	
	[.427]	[.565]	[.310]	[.821]	[.070]	[.971]	

NOTE: These F statistics test zero restrictions on the coefficients of the error correction terms in the error-correction regression for the country/region-specific foreign variables. The figures in square brackets are estimated probability values of the tests.

*Denotes statistical significance at the 5% level or less.

in France and oil prices in the U.S. model. Arguably, the most convincing and plausible of these rejections is the weak exogeneity of oil prices in the U.S. model. So we reestimated the U.S. model with oil prices as endogenous. This did not affect our main conclusion about the number of cointegrating relations in the U.S. model, but did confirm the importance of possible feedback effects from the U.S. economy into oil prices. There seems to be little to choose from between the two versions of the U.S. model, however. After careful consideration of the various issues involved, we decided in favor of treating oil prices as exogenous throughout the GVAR model, considering the importance of geopolitical factors in the determination of oil prices and the desirability of retaining a flexible modeling approach suited to the analysis of special risks from international political events, such as threats of war and terrorism.

9.6 Other Features of the Country-Specific Models

Due to data limitations and the relatively large number of endogenous and exogenous variables involved, we have been forced to consider a VARX$^*(1, 1)$ specifications for the country-specific models. It is therefore important to check the adequacy of the country-specific models in dealing with the complex dynamic interrelationships that exist in the world economy. To this end, Table 8 provides F-statistics for tests of serial correlation of order 4 in the residuals of the error-correction regressions for all of the 63 endogenous variables in the GVAR model.

Considering the relative simplicity of the underlying models, it is comforting that 45 of the 63 regressions pass the residual serial correlation test at the 95% level. Perhaps not surprisingly, most of the statistically significant outcomes occur in the case of variables with known growth persistence characteristics, namely real money balances, interest rates, and inflation. But even in these examples the degree of rejection is not uniform. For example, using the 1% significance level, there are only seven error correction equations that do not meet the requirement. Therefore, although there are cases of concern that need to be examined more carefully, overall the test results seem satisfactory. It is hoped that as more data become available, higher-order VARX* models can be estimated and their results evaluated for residual serial correlation. This may require estimation of different-order VARX* models for different countries. The GVAR methodology can accommodate both extensions, but these will not be pursued here.

These test results, together with the weak exogeneity of the foreign variables, also allow consistent estimation of the contemporaneous effects of foreign-specific variables on domestic variables (at least for the ones where the residual serial correlation test is not statistically significant). There are many estimates of interest that could be considered; here we focus on the

Table 8. F Statistics for Tests of Residual Serial Correlation for the Country-Specific ECM Regressions

Country/region	Domestic variables Δy	$\Delta^2 p$	Δq	$\Delta(e-p)$	$\Delta\rho$	Δm
U.S.						
Serial correlation, F (4, 70)	.97	4.34*	.35		2.72*	.31
	[.43]	[0]	[.84]		[.04]	[.87]
U.K.						
Serial correlation, F (4, 63)	.33	2.68*	1.76	1.70	1.24	.77
	[.86]	[.04]	[.15]	[.16]	[.30]	[.55]
Germany						
Serial correlation, F (4, 64)	1.14	2.32	2.32	.10	2.91*	2.57*
	[.34]	[.07]	[.07]	[.98]	[.03]	[.05]
France						
Serial correlation, F (4, 65)	1.30	1.46	.75	.89	.69	2.57*
	[.28]	[.23]	[.56]	[.48]	[.60]	[.05]
Italy						
Serial correlation, F (4, 65)	1.16	9.39*	1.09	.74	1.80	3.33*
	[.34]	[0]	[.37]	[.57]	[.14]	[.02]
Western Europe						
Serial correlation, F (4, 65)	.95	.61	1.00	.28	1.29	.66
	[.44]	[.66]	[.42]	[.89]	[.28]	[.62]
Middle East						
Serial correlation, F (4, 65)	17.24*	.71		1.44	2.66*	.81
	[0]	[.59]		[.23]	[.04]	[.52]
China						
Serial correlation, F (4, 65)	1.92	1.50		1.82	.41	5.70*
	[.12]	[.21]		[.14]	[.80]	[0]
Southeast Asia						
Serial correlation, F (4, 65)	1.03	3.27*	1.52	3.60*	2.54*	.55
	[.40]	[.02]	[.21]	[.01]	[.05]	[.70]
Japan						
Serial correlation, F (4, 64)	2.88*	2.18	2.08	1.28	2.63*	3.48*
	[.03]	[.08]	[.09]	[.29]	[.04]	[.01]
Latin America						
Serial correlation, F (4, 65)	2.21	2.45	1.81	.47	1.82	3.42*
	[.08]	[.06]	[.14]	[.76]	[.14]	[.01]

NOTE: The figures in square brackets are probability values associated with *F* statistics.
* Denotes statistical significance at the 5% level or less.

contemporaneous effects of the foreign variables on their domestic counterpart. For example, we could ask about the effect on German output if foreign output specific to Germany rises by 1%. Similarly, the effect of 1% increase in "world" equity prices on equity prices of the individual countries/regions can be estimated. The estimates, best viewed as impact elasticities, from such an exercise are summarized in Table 9. When statistically significant, all of the estimates have the expected sign of

Table 9. Contemporaneous Effects of Foreign Variables on Their Domestic Counterparts in Country-Specific Models

Country/region	Domestic variables y	Δp	q	ρ	m
U.K.	.2099	.2493	.6582*	.2362	−.7759
	(.1658)	(.2316)	(.0862)	(.1401)	(.8144)
Germany	.9293*	.1955	.9297*	.1556*	.5714*
	(.2352)	(.1569)	(.1229)	(.0389)	(.1571)
France	.5112*	.3381*	.7530*	.2909*	−.1141
	(.1547)	(.1409)	(.1202)	(.1191)	(.1057)
Italy	.6135*	−.3075	1.4386*	.2226*	.5884*
	(.1497)	(.2171)	(.2227)	(.0637)	(.1996)
Western Europe	.3430*	.2109*	.6165*	.0627	.3325*
	(.0844)	(.0986)	(.1395)	(.0680)	(.1303)
Middle East	.0546	1.4986		.4887	.1919
	(.1876)	(.9462)		(.6460)	(.1419)
China	.8020*	.1274		.0023	.4637
	(.2235)	(.2064)		(.0446)	(.8578)
Southeast Asia	−.0550	.4635*	.7168*	.1108	−.4936*
	(.1498)	(.2978)	(.1798)	(.2008)	(.2070)
Japan	.5354*	.0830	.5031*	−.1341*	.0151
	(.1685)	(.1069)	(.1682)	(.0359)	(.1366)
Latin America	.2340	6.4410*	1.2429*	−3.7111	2.1383*
	(.1434)	(2.8000)	(.3758)	(8.5494)	(.7986)

NOTE: The figures in parentheses are standard errors of the estimates.
* Denotes statistical significance at the 5% level or less.

being positive, except for the coefficients of Δm^* in Southeast Asia and $\Delta\rho^*$ in Japan.

The output elasticities are significant in the case of Germany, France, Italy, Western Europe, China, and Japan. Equity price elasticities are statistically significant in the case of all countries/regions with a capital market. The patterns of statistical significance of inflation, interest rate, and real money balances are more dispersed across countries. Perhaps not surprisingly, equity markets show the closest degree of contemporaneous interdependence, with the other channels playing a less prominent role in comparison.

9.7 Dynamic Properties of the Global Model

Due to the simultaneous nature of the country-specific models, a more satisfactory approach to the analysis of dynamics and interdependencies (both on impact and over time) among the various factors would be via impulse response functions computed from the solution to the GVAR model. As discussed in Section 3, the global model can be obtained by combining the country-specific models. The total number of cointegrating relations in the global model can be at most equal to $r = \sum_{i=0}^{10} r_i = 36$. The long-run and short-run dynamic properties of the global model are determined by the global cointegrating matrix, $\tilde{\boldsymbol{\beta}}$, given by (21), and the eigenvalues of $\boldsymbol{F} = \mathbf{G}^{-1}\mathbf{H}$, defined by (25). Because the global model contains 63 endogenous variables and the rank of $\tilde{\boldsymbol{\beta}}$ is at most 36, it then follows that $\boldsymbol{F}$ must have at least 27 (63 − 36) eigenvalues that fall on the unit circle. It is encouraging that our application does in fact satisfy this property. The matrix $\boldsymbol{F}$, estimated from the region-specific models, has exactly 27 eigenvalues that fall on the unit circle, with the remaining 36 eigenvalues having moduli all less than unity. Of these 36 eigenvalues, 28 (14 pairs) were complex, producing the damped, mildly cyclical character of the generalized impulse response functions discussed later. The eigenvalues with the three largest complex parts are $.3875 \pm .2495i$, $.1023 \pm .1990i$, and $.7406 \pm .1624i$, where $i = \sqrt{-1}$. Apart from the unit roots, the three largest eigenvalues (in moduli) are .9456, .8661, and .8575, thus ensuring a reasonably fast rate of convergence of the model to its steady state once shocked. These results also establish that the global model forms a cointegrating system with 36 long-run relations and a stable error-correcting representation. In particular, the effects of shocks on the long-run relations of the global economy will eventually disappear. The decay rate is bounded by .9456. However, due to the unit root properties of the global model (as characterized by the unit eigenvalues of $\boldsymbol{F}$), global or regional shocks will have permanent effects on the levels of the variables such as real outputs, interest rates, or real equity prices.

The time profiles of the effects of various shocks of interest on the global economy can now be computed using the GIRFs discussed in Section 6, which identify the shocks as intercept shifts in the various equations using a historical variance–covariance matrix of the errors. This approach is particularly suited for the analysis of dynamics of the transmission of shocks across regions, because GIRFs are invariant to the ordering of the countries/regions in the GVAR model (see Sec. 6). Also, although it is true that it may not be possible to provide "structural" *economic* interpretation of these shocks as "demand," "supply," or "policy" shocks, GIRFs provide a historically consistent account of the interdependencies of the idiosyncratic shocks, particularly across different regions. Given that specific-country models condition on weakly exogenous foreign variables, it is reasonable to expect that there should remain only a modest degree of correlations across the shocks from different regions, and hence it is more reasonable to believe that the GVAR helps identify regional shocks as compared with shocks that can be given a satisfactory economic interpretations. For example, the GVAR approach could provide a plausible account of the transmission of shocks from the U.S. (modeled almost as a closed economy) to the rest of the world. Accordingly, we consider the following shock scenarios with emphasis on their regional transmissions:

- A 1 standard error negative shock (a negative "unit" shock) to U.S. equity prices
- A 1 standard error positive shock to German output
- A 1 standard error negative shock to equity markets in Southeast Asia.

We could examine the time profiles of the effects of these shocks either on the endogenous variables of a particular region or on a given variable across all of the regions.

9.7.1 A Negative Shock to U.S. Equity Prices. Figure 1 displays the impacts of shocks to U.S. equity market on equity prices worldwide.

On impact, a fall in U.S. equity prices causes prices in all equity markets to fall as well, but by smaller amounts: 3.5% in the U.K., 4.5% in Germany, 2.4% in Japan, 2.6% in Southeast Asia, and 4.8% in Latin America, compared with a fall of 6.4% in the U.S. (see Table 10).

However, over time the fall in equity prices across the regions start to catch up with the U.S. and gets amplified in the case of Italy and Latin America. The U.K. presents an interesting exception to this pattern, although these point estimates should be viewed with caution. They are likely to be poorly estimated with large standard errors, particularly those that refer to long forecast horizons. [It is possible to compute standard errors for the generalized impulse responses using bootstrap techniques;

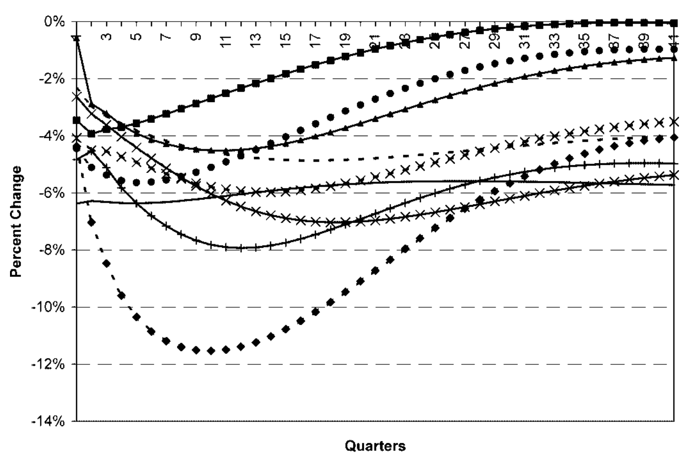

Figure 1. Impulse Response of a Negative Unit (−1σ) Shock to U.S. Real Equity Prices on Real Equity Prices Across Regions (—— U.S.; —■— U.K.; - ● - Germany; - × - France; - ◆ - Italy; —▲— Western Europe; —×— Southeast Asia; - - - - - Japan; —+— Latin America).

Table 10. Generalized Impulse Responses of a Negative One Standard Error Shock to U.S. Equity Prices

Region	*Quarters after shock* 0	1	2	3	4	8	12	20
On real equity prices (%)								
U.S.	−6.38	−6.28	−6.32	−6.35	−6.36	−6.22	−5.98	−5.65
U.K.	−3.46	−3.93	−3.78	−3.70	−3.57	−2.87	−2.16	−.96
Germany	−4.45	−5.11	−5.37	−5.57	−5.64	−5.28	−4.49	−2.72
France	−4.07	−4.51	−4.54	−4.73	−4.93	−5.66	−5.97	−5.45
Italy	−4.37	−7.03	−8.47	−9.59	−10.35	−11.51	−11.23	−8.72
Western Europe	−.57	−2.88	−3.21	−3.62	−3.92	−4.48	−4.44	−3.40
Middle East								
China								
Southeast Asia	−2.63	−3.23	−3.62	−4.03	−4.42	−5.76	−6.65	−6.96
Japan	−2.35	−2.95	−3.30	−3.58	−3.82	−4.46	−4.78	−4.79
Latin America	−4.83	−4.53	−5.12	−5.82	−6.36	−7.67	−7.91	−6.73
On real output (%)								
U.S.	−.10	−.23	−.27	−.29	−.31	−.32	−.29	−.22
U.K.	−.11	−.19	−.23	−.27	−.29	−.27	−.17	.02
Germany	.06	−.10	−.15	−.20	−.25	−.45	−.61	−.76
France	.01	.01	0	−.03	−.06	−.20	−.29	−.34
Italy	.01	−.08	−.13	−.18	−.23	−.36	−.44	−.44
Western Europe	−.09	−.14	−.21	−.27	−.32	−.48	−.58	−.60
Middle East	−.07	−.03	0	.03	.05	.09	.06	−.03
China	.27	.36	.43	.46	.47	.41	.32	.25
Southeast Asia	−.01	−.04	−.06	−.09	−.12	−.28	−.43	−.57
Japan	.13	.14	.15	.14	.11	−.05	−.16	−.28
Latin America	0	−.06	−.13	−.20	−.26	−.47	−.56	−.50
On inflation (%)								
U.S.	.06	−.06	−.09	−.09	−.10	−.11	−.11	−.10
U.K.	.05	.03	−.03	−.03	−.05	−.09	−.11	−.12
Germany	.05	.01	.01	0	0	−.03	−.05	−.07
France	−.05	−.01	.01	.01	.01	0	0	0
Italy	−.02	.03	.07	.09	.10	.09	.07	.03
Western Europe	.05	−.03	−.01	−.01	−.01	0	.01	.01
Middle East	.60	.22	.22	.17	.14	−.01	−.09	−.15
China	.01	.04	.09	.12	.15	.21	.21	.17
Southeast Asia	−.09	−.09	−.08	−.08	−.08	−.09	−.11	−.13
Japan	−.05	−.04	−.06	−.06	−.07	−.08	−.09	−.10
Latin America	1.05	1.34	.93	1.06	1.17	1.43	1.57	1.57
On interest rate (%)								
U.S.	−.01	−.06	−.07	−.06	−.06	−.07	−.08	−.10
U.K.	−.04	−.04	−.07	−.08	−.10	−.12	−.12	−.10
Germany	.01	0	−.02	−.03	−.05	−.10	−.15	−.21
France	.01	.01	0	0	0	−.01	−.02	−.03
Italy	.02	.04	.05	.06	.06	.06	.05	.02
Western Europe	−.04	−.06	−.07	−.07	−.08	−.08	−.08	−.08
Middle East	.15	.12	.17	.20	.23	.30	.34	.38
China	−.01	0	.01	.02	.03	.05	.06	.05
Southeast Asia	−.04	−.02	−.01	−.01	0	0	−.01	−.03
Japan	.02	.02	.01	0	−.01	−.04	−.05	−.07
Latin America	1.61	1.09	1.10	1.24	1.34	1.60	1.72	1.67
On real exchange rate (%)								
U.K.	−.70	−.77	−.61	−.37	−.03	1.28	2.02	2.20
Germany	−.99	−1.06	−.99	−.86	−.68	.33	1.36	2.88
France	−.34	−.74	−.56	−.37	−.17	.49	.86	1.02
Italy	−.10	.32	.84	1.19	1.51	2.50	3.02	3.01
Western Europe	−.70	−.38	.09	.51	.91	2.21	3.02	3.41
Middle East	−.49	−.93	−1.14	−1.33	−1.50	−1.77	−1.63	−1.01
China	1.00	.33	−.16	−.51	−.80	−1.40	−1.47	−1.15
Southeast Asia	.23	.12	.13	.13	.14	.29	.41	.51
Japan	−.32	−.37	−.47	−.49	−.45	−.20	−.01	.17
Latin America	.56	.68	.87	1.13	1.37	2.13	2.51	2.41
On real money supply (%)								
U.S.	−.11	−.14	−.18	−.21	−.23	−.21	−.12	.04
U.K.	−.60	−.50	−.50	−.53	−.57	−.77	−.83	−.45
Germany	−.13	−.02	−.03	−.06	−.10	−.26	−.38	−.48
France	−.27	−.41	−.56	−.69	−.79	−.97	−.96	−.72
Italy	.02	.28	.49	.62	.69	.75	.72	.58
Western Europe	−.14	−.08	−.20	−.27	−.34	−.54	−.63	−.61
Middle East	.19	.06	−.09	−.21	−.32	−.59	−.69	−.62
China	−.58	−.25	−.22	−.28	−.39	−.88	−1.07	−.85
Southeast Asia	−.18	−.11	−.09	−.09	−.10	−.16	−.23	−.29
Japan	−.07	−.17	−.28	−.38	−.46	−.67	−.79	−.88
Latin America	.22	−.32	−.62	−.84	−1.11	−2.08	−2.50	−2.18

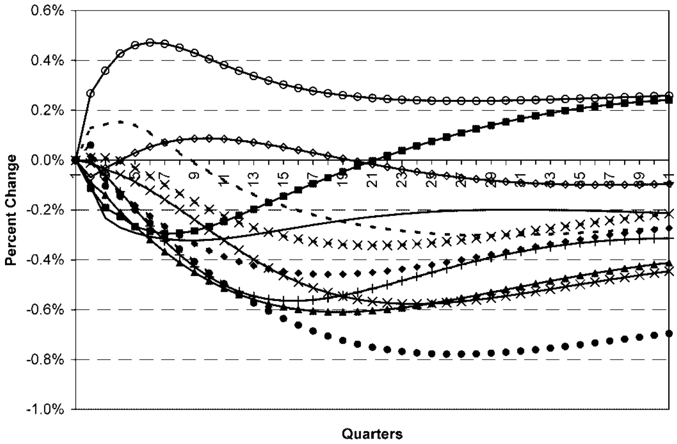

Figure 2. Impulse Response of a Negative (−1σ) Shock to U.S. Real Equity Prices on Real Output Across Regions (—— U.S.; —■— U.K.; - ●- Germany; - ×- France; - ◆- Italy; —▲— Western Europe; —◇— Middle East; —○— China; —×— Southeast Asia; ----- Japan; —+— Latin America).

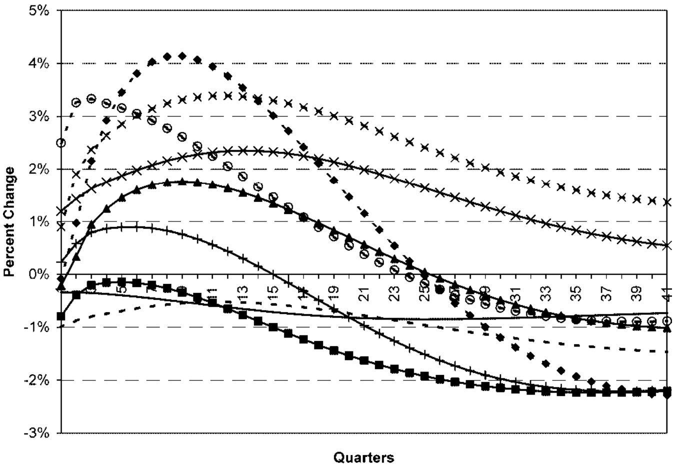

Figure 3. Impulse Response of a Positive Unit (−1σ) Shock to German Real Output on Real Equity Prices Across Regions (—— U.S.; —■— U.K.; - ○- Germany; - ×- France; - ◆- Italy; —▲— Western Europe; —×— Southeast Asia; ---- Japan; —+— Latin America).

see, e.g., Garratt et al. (2003a). This would be a highly computer intensive exercise, and it is not clear to us that it would add much to our overall conclusions.] Nevertheless, the relative position and pattern of the impulse response functions could still be quite informative. For example, they confirm the pivotal role played by the U.S. stock market in the global economy, and suggest that in the longer run scope for geographic diversifications across equity market might be somewhat limited (see Fig. 1).

The time profiles of the effects of the shock to the U.S. equity market on real output across the different regions are shown in Figure 2. The second panel of Table 10 provides the associated point estimates for a number of selected horizons.

The impact response of the fall in the U.S. equity market on real output is negative for most regions, but rather small in magnitude. After one year real output shows a fall of around .31% in the U.S., .25% in Germany, .29% in the U.K., .26% in Latin America, and .12% in Southeast Asia. Japanese output only begins to be negatively affected by the adverse U.S. stock market shock much later. The two regions without capital markets are either not affected by the shock (Middle East) or even show a rise in output (in the case of China). Once again, these point estimates should be treated with caution.

Table 10 also provides point estimates of the time profiles of the effects of the adverse U.S. stock market shock on inflation, interest rates, and real exchange rates. Overall, the pattern of the impulse responses across the regions seems plausible, although space does not permit a detailed discussion of these results here.

9.7.2 A Positive Shock to German Output. The effects of a one standard error rise in output in Germany on equity prices and real output across the different regions are summarized in Table 11 and displayed in Figures 3 and 4. A one standard error shock here converts to an approximate 2.96% per annum increase in output.

Table 11 also provides the point estimates of the effects of the shock on inflation, interest rates, and real exchange rates for selected horizons. On impact, the effect of the increase in Germany's output is to increase German equity prices by 2.50%, followed by 1.20% in Southeast Asia and .90% in France, with mixed outcomes for the remaining regions. The impact effects are in fact negative on U.S., U.K., and Japan equity prices, although these are rather small compared with standard errors of shocks to equity prices in these economies. Over time, the effect of the positive output shock is to increase equity prices in France, Italy, and the rest of Western Europe in line with the increase in Germany's equity prices, although the effects on U.K. and U.S. equity prices continue to remain negative but very small. This shows the high degree of integration of the European economies with Germany with the notable exception of the U.K. equity market, which seems to follow the U.S. market instead.

A similar story also emerges if the effects of the shock to Germany's outputs on other countries' outputs are considered (see Table 11). After one year, the effect of the shock on U.S. and U.K. output is almost zero but is still sizeable on France, Italy, and the rest of Western Europe. These differences become further pronounced at horizons beyond one year.

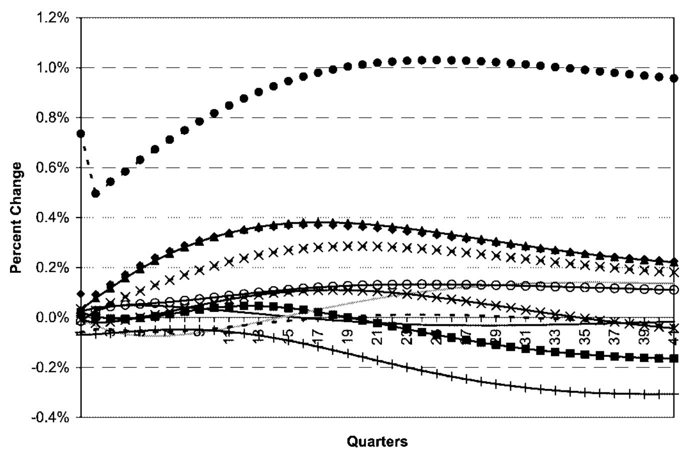

Figure 4. Impulse Response of a Positive (−1σ) Shock to German Real Output on Real Output Across Regions (—— U.S.; —■— U.K.; - ●- Germany; - ×- France; - ◆- Italy; —▲— Western Europe; ——— Middle East; —○— China; —×— Southeast Asia; ----- Japan; —+— Latin America).

Table 11. Generalized Impulse Responses of a Positive One Standard Error Shock to Germany Output

	Quarters after shock							
Region	*0*	*1*	*2*	*3*	*4*	*8*	*12*	*20*
On real equity prices (%)								
U.S.	−.34	−.34	−.35	−.37	−.39	−.52	−.65	−.83
U.K.	−.79	−.39	−.20	−.15	−.14	−.34	−.77	−1.63
Germany	2.50	3.26	3.33	3.24	3.15	2.61	1.86	.38
France	.90	1.90	2.36	2.64	2.85	3.31	3.37	2.82
Italy	−.09	.97	2.15	2.93	3.46	4.14	3.54	1.15
Western Europe	−.21	.34	.94	1.24	1.46	1.76	1.55	.56
Middle East								
China								
Southeast Asia	1.20	1.43	1.62	1.75	1.87	2.23	2.35	1.99
Japan	−.99	−.87	−.80	−.74	−.69	−.55	−.53	−.78
Latin America	.23	.58	.80	.87	.89	.76	.30	−.97
On real output (%)								
U.S.	.02	.04	.05	.05	.05	.04	.01	−.02
U.K.	.02	0	0	−.01	0	.03	.05	−.02
Germany	.74	.50	.54	.58	.63	.78	.90	1.02
France	.03	.03	.06	.08	.11	.19	.25	.28
Italy	.09	.09	.13	.17	.21	.31	.36	.36
Western Europe	.03	.08	.12	.16	.19	.30	.36	.37
Middle East	−.01	−.03	−.06	−.07	−.07	−.06	−.02	.08
China	−.01	.03	.04	.05	.05	.07	.10	.13
Southeast Asia	−.01	−.02	−.02	−.01	0	.05	.09	.10
Japan	−.06	−.05	−.05	−.05	−.06	−.05	−.03	.01
Latin America	−.07	−.07	−.06	−.06	−.05	−.05	−.07	−.17
On inflation (%)								
U.S.	−.03	−.02	−.01	−.01	−.01	−.01	−.01	−.02
U.K.	.05	.06	.06	.05	.05	.07	.08	.09
Germany	−.12	.04	.01	.01	.02	.04	.05	.07
France	−.07	−.05	−.04	−.05	−.06	−.09	−.10	−.11
Italy	.06	.05	.03	.03	.02	.03	.05	.09
Western Europe	−.03	−.02	−.01	−.02	−.02	−.03	−.03	−.03
Middle East	−.26	−.05	−.03	−.04	−.02	.03	.08	.09
China	−.06	−.02	0	0	0	0	.01	.03
Southeast Asia	.08	.05	.03	.03	.03	.03	.03	.02
Japan	.01	.01	.01	.01	.01	.01	.02	.01
Latin America	.24	.10	.04	−.03	−.07	−.13	−.15	−.09
On interest rate (%)								
U.S.	.01	.02	.02	.02	.02	.03	.03	.04
U.K.	.04	.04	.04	.03	.03	.04	.04	.04
Germany	.02	.02	.04	.05	.06	.11	.15	.19
France	−.02	−.02	−.02	−.02	−.02	−.02	−.01	−.01
Italy	.01	.01	0	0	0	.02	.03	.07
Western Europe	−.01	0	0	.01	.01	.01	.01	0
Middle East	−.09	.03	.06	.06	.05	.02	0	−.02
China	.01	.01	.01	.01	.01	.01	.01	.02
Southeast Asia	.02	.01	0	0	0	0	0	.01
Japan	0	−.01	−.02	−.02	−.03	−.04	−.04	−.03
Latin America	0	.02	−.07	−.11	−.13	−.18	−.18	−.09
On real exchange rate (%)								
U.K.	.02	−.04	−.47	−.75	−.95	−1.38	−1.64	−1.71
Germany	.21	.11	−.22	−.51	−.75	−1.67	−2.50	−3.68
France	.14	.37	.50	.55	.56	.45	.31	.30
Italy	−.49	−.78	−1.06	−1.40	−1.69	−2.45	−2.79	−2.72
Western Europe	−.16	−.53	−.90	−1.26	−1.58	−2.52	−3.06	−3.30
Middle East	−.89	−.51	−.34	−.26	−.20	−.26	−.53	−1.21
China	.73	.83	.86	.86	.88	.89	.81	.51
Southeast Asia	.47	.33	.29	.27	.26	.22	.17	.12
Japan	.42	.49	.50	.51	.51	.49	.43	.34
Latin America	−.18	−.16	−.19	−.22	−.24	−.28	−.24	.06
On real money supply (%)								
U.S.	.19	.20	.21	.21	.20	.16	.10	.02
U.K.	−.88	−.46	−.15	.15	.41	1.06	1.29	1.11
Germany	.55	.89	.94	.99	1.02	1.13	1.20	1.26
France	.19	.16	.11	.07	.04	−.05	−.16	−.43
Italy	.35	.39	.27	.19	.14	.08	.12	.26
Western Europe	.14	.27	.32	.38	.44	.57	.63	.58
Middle East	.11	.08	.09	.12	.16	.24	.25	.13
China	.46	.45	.45	.47	.48	.51	.45	.20
Southeast Asia	.31	.29	.29	.29	.30	.31	.32	.32
Japan	.01	−.01	−.01	−.01	0	.04	.07	.07
Latin America	.11	0	−.13	−.17	−.18	−.24	−.42	−1.01

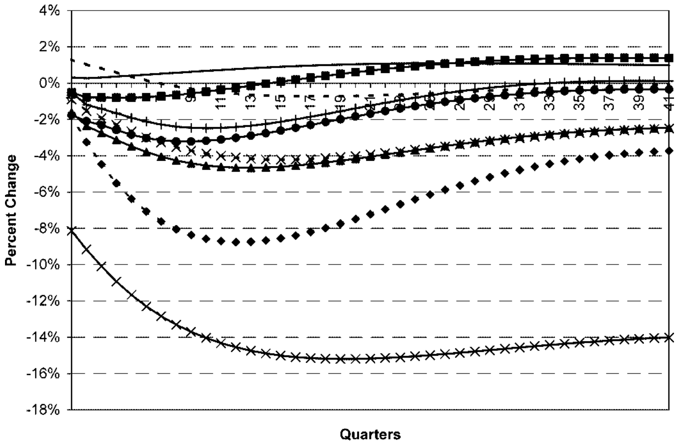

Figure 5. Impulse Response of a Negative Unit (−1σ) Shock to Southeast Asian Real Equity Prices on Real Equity Prices Across Regions (—— U.S.; —■— U.K.; —●— Germany; - ×- France; - ◆- Italy; —▲— Western Europe; —×— Southeast Asia; ----- Japan; —+— Latin America).

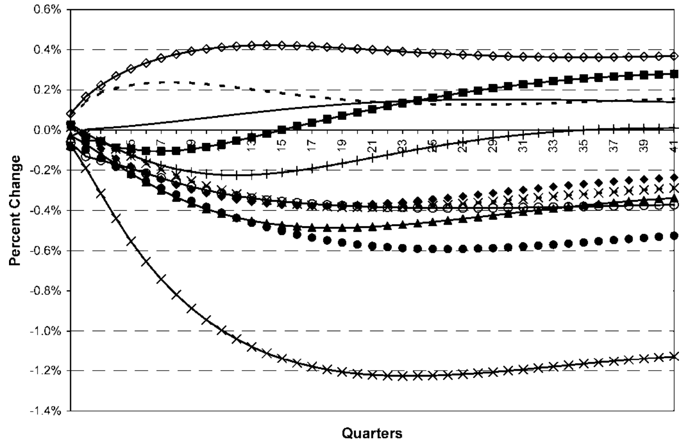

Figure 6. Impulse Response of a Negative (−1σ) Shock to Southeast Asian Real Equity Prices on Real Output Across Regions (—— U.S.; —■— U.K.; - ●- Germany; - ×- France; - ◆- Italy; —▲— Western Europe; —◇— Middle East; —○— China; —×— Southeast Asia; ---- Japan; —+— Latin America).

The effects of the shock on other variables are mixed. They are mostly small and transient in the case of inflation and interest rates, but quite sizeable as far as exchange rate and real money balances are concerned, at least in the case of some of the countries, notably Germany, Italy, and the rest of Western Europe.

9.7.3 A Negative Shock to Equity Markets in Southeast Asia. Given the interest in the effects of the 1997 Southeast Asian Crisis and its possible contagion effects, we consider here the GIRFs for a one standard error negative shock to equity prices in Southeast Asia. The one standard error shock is equivalent to a 8.2% decline in Southeast Asia's equity prices and has a small positive effect on Japan and U.S. equity prices (1.30% and .31%) and relatively small negative effects on equity prices in other countries (see Table 12 and Fig. 5).

But over time, these effects accumulate, and after two years all markets are adversely affected with the exception of the U.S.. The U.S. equity market (and to a lesser extent, the U.K. and Japanese markets) seem to have been reasonably robust to the Southeast Asian Crisis. It is also interesting to note that in the longer run, the Western European (except for the U.K.) equity markets seem to be more vulnerable to the Southeast Asian Crisis than Japan.

As to be expected, the output effects of the negative shock to the Southeast Asia equity markets are much more muted than its effects on equity prices. Even after one year, adverse effects of the shock is sizeable only in the case of European economies (with the exception of U.K.), with the largest effect, perhaps not surprisingly, on Southeast Asia itself (see Table 12 and Fig. 6).

Once again, the impulse responses suggest that Japan, the U.S., and the U.K. are likely to be reasonably robust to adverse shocks from Southeast Asian equity markets. At first, this result seems rather surprising, considering the relatively strong trade links that exists between Southeast Asia, Japan, and the U.S. (see Table 2). However, this result largely reflects the apparently weak links that exist between the equity markets of these economies, as can be seen Table 12 and discussed earlier. The impulse responses of the effects of the negative shock to Southeast Asia equity markets on inflation, interest rates, and exchange rates are summarized in Table 12. Other implications of the Southeast Asian Crisis (such as an adverse shock to exchange rates) can also be investigated using the GVAR modeling tools developed in this article.

10. CONDITIONAL LOSS DISTRIBUTIONS

To illustrate some of the ideas and applications of the GVAR, in this section we show how to use the model to generate conditional loss distributions. Here we summarize and report a simplified version of the conditional credit risk model developed in detail by Pesaran, Schuermann, Treutler, and Weiner (2003), hereafter denoted by PSTW. (For full details of the technical derivations, particularly for multiperiod loss forecasting, consult PSTW.)

We begin with a characterization of a firm's change in value as a function of systematic and idiosyncratic components. Following an approach that is structurally similar to arbitrage pricing theory (APT), a firm's change in value (or return), conditional on information available up to time t, $\mathcal{I}_t$, can be decomposed as

$$r_{ji,t+1} = \mu_{jit} + \xi_{ji,t+1}, \tag{66}$$

where μ_{jit} is the (forecastable) conditional mean and $\xi_{ji,t+1}$ is the (nonforecastable) innovation component of the return process of the jth firm in the ith country/region. Consistent with the distributional assumptions of the GVAR model, the innovation has a conditional Gaussian distribution,

$$\xi_{ji,t+1}|\mathcal{I}_t \sim \mathrm{N}(0, \omega^2_{\xi,ji}). \tag{67}$$

The normality assumption could be a good approximation for quarterly returns, but it is relatively easy to adapt the analysis to allow for fat-tailed distributions, such as Student t, with low degrees of freedom; in fact, this is done in PSTW. The assumption that the conditional variance of returns are time invariant also seems reasonable for quarterly returns, although it would need to be relaxed for returns measured over shorter periods, such as weeks or days.

Table 12. Generalized Impulse Responses of a Negative One Standard Error Shock to Southeast Asia Equities

	Quarters after shock							
Region	*0*	*1*	*2*	*3*	*4*	*8*	*12*	*20*
On real equity prices (%)								
U.S.	.31	.27	.31	.37	.42	.62	.82	1.06
U.K.	−.52	−.78	−.79	−.80	−.80	−.57	−.14	.70
Germany	−1.75	−2.10	−2.31	−2.58	−2.84	−3.21	−2.89	−1.68
France	−.90	−1.53	−1.92	−2.29	−2.66	−3.72	−4.17	−3.93
Italy	−1.79	−3.25	−4.47	−5.51	−6.36	−8.36	−8.74	−7.22
Western Europe	−1.62	−2.36	−2.74	−3.13	−3.48	−4.42	−4.66	−4.05
Middle East								
China								
Southeast Asia	−8.15	−9.16	−10.09	−10.93	−11.66	−13.70	−14.74	−15.17
Japan	1.30	1.03	.82	.58	.36	−.31	−.64	−.68
Latin America	−.53	−1.19	−1.41	−1.65	−1.90	−2.46	−2.35	−1.23
On real output (%)								
U.S.	−.03	0	.01	.01	.02	.05	.09	.14
U.K.	.03	−.05	−.06	−.09	−.09	−.09	−.04	.10
Germany	−.09	−.10	−.14	−.18	−.22	−.35	−.46	−.58
France	.01	−.02	−.05	−.09	−.13	−.25	−.33	−.38
Italy	.03	−.04	−.09	−.14	−.18	−.29	−.36	−.37
Western Europe	−.03	−.07	−.12	−.17	−.22	−.37	−.45	−.48
Middle East	.08	.17	.22	.27	.30	.39	.42	.40
China	−.06	−.13	−.15	−.17	−.19	−.28	−.34	−.38
Southeast Asia	−.07	−.19	−.32	−.44	−.55	−.89	−1.08	−1.22
Japan	.07	.14	.19	.21	.22	.23	.20	.14
Latin America	.03	−.02	−.06	−.09	−.12	−.21	−.22	−.14
On inflation (%)								
U.S.	−.06	−.01	−.01	−.01	−.01	0	0	.01
U.K.	−.11	−.01	−.08	−.06	−.09	−.12	−.14	−.15
Germany	.01	−.03	−.02	−.03	−.03	−.05	−.07	−.08
France	.01	−.02	−.03	−.03	−.03	−.05	−.06	−.06
Italy	.03	.02	.03	.04	.05	.07	.07	.04
Western Europe	0	−.04	−.03	−.03	−.03	−.03	−.02	−.02
Middle East	.60	.03	.10	.06	.03	−.11	−.21	−.27
China	.06	.04	.05	.06	.06	.05	.03	0
Southeast Asia	−.15	−.12	−.11	−.12	−.15	−.24	−.29	−.32
Japan	−.09	−.05	−.05	−.06	−.06	−.09	−.10	−.11
Latin America	−.27	.21	.27	.27	.29	.40	.45	.41
On interest rate (%)								
U.S.	0	.03	.03	.02	.02	.01	0	−.01
U.K.	−.01	−.01	−.03	−.03	−.04	−.07	−.07	−.06
Germany	−.03	−.03	−.03	−.04	−.05	−.09	−.12	−.16
France	−.03	−.03	−.03	−.03	−.04	−.05	−.07	−.08
Italy	.02	.02	.03	.03	.03	.04	.04	.02
Western Europe	−.01	−.01	−.01	−.01	−.02	−.03	−.03	−.03
Middle East	.07	−.15	−.14	−.14	−.13	−.10	−.07	−.04
China	−.01	0	0	0	.01	.01	0	−.01
Southeast Asia	.06	.03	.02	.01	.01	−.02	−.03	−.05
Japan	−.03	−.03	−.02	−.01	0	0	0	−.02
Latin America	.02	.25	.25	.28	.31	.42	.46	.39
On real exchange rate (%)								
U.K.	−.52	−.57	−.20	.02	.29	1.21	1.83	2.07
Germany	−1.59	−1.48	−1.42	−1.41	−1.41	−1.02	−.37	.74
France	−1.33	−1.30	−1.17	−1.04	−.91	−.37	0	.22
Italy	−1.08	−.61	−.15	.21	.48	1.25	1.66	1.70
Western Europe	−1.36	−1.11	−.80	−.50	−.22	.82	1.51	1.90
Middle East	−.17	−.81	−1.20	−1.53	−1.81	−2.52	−2.74	−2.56
China	−1.49	−1.44	−1.54	−1.67	−1.80	−2.04	−2.02	−1.80
Southeast Asia	.40	1.14	1.47	1.57	1.58	1.52	1.52	1.55
Japan	−.90	−.71	−.66	−.63	−.62	−.55	−.46	−.34
Latin America	.29	.49	.62	.75	.87	1.21	1.31	1.07
On real money supply (%)								
U.S.	.12	.10	.12	.13	.15	.24	.32	.44
U.K.	.65	.64	.60	.53	.46	.26	.18	.41
Germany	−.08	−.19	−.21	−.22	−.23	−.30	−.37	−.45
France	−.04	−.14	−.23	−.31	−.39	−.63	−.71	−.59
Italy	−.20	−.12	−.01	.08	.15	.29	.31	.24
Western Europe	−.28	−.27	−.34	−.40	−.45	−.64	−.74	−.74
Middle East	−.16	−.18	−.28	−.37	−.46	−.75	−.90	−.94
China	−.17	−.46	−.66	−.80	−.93	−1.25	−1.28	−1.04
Southeast Asia	−.12	−.33	−.44	−.53	−.60	−.79	−.89	−.97
Japan	.02	.05	.05	.05	.04	−.03	−.10	−.15
Latin America	.32	0	−.36	−.61	−.79	−1.22	−1.27	−.78

Linking the firm return expression (66) into the GVAR model, we can specify the conditional mean process more precisely. Thus firm returns depend on changes in the underlying domestic macroeconomic factors, say k_i region-specific macroeconomic variables, the exogenous global variables ($\mathbf{d}_t$ in our application oil prices), together comprising the systematic components and the firm-specific idiosyncratic shocks,

$$r_{ji,t+1} = \alpha_{ji} + \sum_{\ell=1}^{k_i} \beta_{ji,\ell} \Delta x_{i,t+1,\ell} + \sum_{\ell=1}^{s} \gamma_{ji,\ell} \Delta d_{t+1,\ell} + \eta_{ji,t+1}, \qquad t = 1, 2, \ldots, T, \quad (68)$$

where $r_{ji,t+1}$ is the equity return from t to $t+1$ for firm j ($j = 1, \ldots, nc_i$) in region i, α_{ji} is a regression constant for company j in region i, k_i is the number of domestic macroeconomic factors (drivers) in region i, $\beta_{ji,\ell}$ is the factor loading corresponding to the change in the ℓth macroeconomic variable for company j in region i, $\Delta x_{i,t+1,\ell}$ is the log difference of the ℓth macroeconomic factor in region i, $d_{t+1,\ell}$ is the ℓth global factor, $\gamma_{ji,\ell}$ is its associated coefficient, and $\eta_{ji,t+1}$ is a firm-specific shock. This can be written more compactly as

$$r_{ji,t+1} = \alpha_{ji} + \boldsymbol{\beta}'_{ji} \Delta \mathbf{x}_{i,t+1} + \boldsymbol{\gamma}'_{ji} \Delta \mathbf{d}_{t+1} + \eta_{ji,t+1}, \quad (69)$$

where $\mathbf{x}_{i,t+1}$ and $\mathbf{d}_{t+1}$ are the $k_i \times 1$ and $s \times 1$ vectors of macroeconomic and global factors, which are precisely the variables in the country-specific models defined by (1) or (22). The main advantage of using the GVAR as a driver for a credit portfolio model is that it provides the correlation structure among macroeconomic variables of the global economy. If the model captures all systematic risk, then the idiosyncratic risk components of any two companies in the model should be uncorrelated.

Accordingly, we assume that the firm-specific shocks, $\eta_{ji,t+1}$, have mean zero, a constant (time-invariant) variance $\omega^2_{\eta,ji}$, are serially uncorrelated, and are distributed independently of the macroeconomic factors. Further, for simulation of the loss distribution, we assume that these shocks are also independently distributed across firms as normal variates, namely $\eta_{ji,t+1} \backsim \text{IIN}(0, \omega^2_{\eta,ji})$.

Relaxing the distributional assumption for $\eta_{ji,t+1}$ is no more difficult than it is for $\xi_{ji,t+1}$. Another alternative is to sample directly from the actual APT regression residuals $\hat{\eta}_{jit}$, assuming that the available sample periods across the different companies are sufficiently large. In our application we have at most 80 data points per company, and for some of the companies in the loan portfolio we have considerably less, and so resampling does not promise to provide a more accurate picture of the true distribution of the residuals; see also the discussion in Section 10.3.

In any given time period, the probability of default for firm j in region i will be correlated, through the influence of common macro effects (or systematic risk factors) in region i, and globally, with the probability of default of other firms in the bank's portfolio. Most credit portfolio models share this linkage of systematic risk factors to default and loss; they differ in specifically how they are linked (Fig. 7) (for detailed comparisons, see Koyluoglu and Hickman 1998; Crouhy, Galai, and Mark 2000; Gordy 2000; and Saunders and Allen 2002).

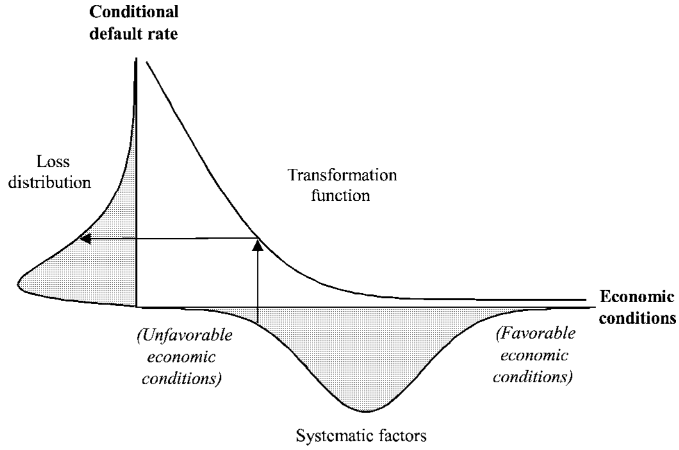

Figure 7. A General Framework for Credit Risk Models. [Adapted with permission from Koyluoglu and Hickman (1998).]

10.1 The Merton Model, Default Thresholds, and Credit Ratings

Before expected loss due to default can be computed, we need a procedure for determining a default threshold, c_{ji}, with respect to which the default state can be defined. The basic premise is that the underlying asset value evolves over time (e.g., through a simple diffusion process), and that default is triggered by a drop in firm's asset value below the value of its callable liabilities. Following Merton (1974), the lender is effectively writing a put option on the assets of the borrowing firm. If the value of the firm falls below a certain threshold, then the shareholders will put the firm to the debt holders. We follow a typical adaptation of the Merton model by using asset returns and their volatility instead of total value of assets and their volatility. But because asset returns and their volatility are difficult to observe directly, we use equity returns and their volatility as proxies.

In the Merton model, default occurs if the value of the firm j in region i at time t falls below a given fixed threshold value, c_{ji}. The separation between a default state and a nondefault state can now be characterized using the indicator variable $I(r_{ji,t+1} < c_{ji})$ such that

$$\begin{aligned} I(r_{ji,t+1} < c_{ji}) &= 1 \quad \text{if} \quad r_{ji,t+1} < c_{ji} \implies \text{default}, \\ I(r_{ji,t+1} < c_{ji}) &= 0 \quad \text{if} \quad r_{ji,t+1} \geq c_{ji} \implies \text{no default}. \end{aligned} \quad (70)$$

In standard implementations of the Merton model, the percentage changes in asset value are taken to be normally distributed. Moreover, this class of models places a specific interpretation on credit ratings from rating agencies, namely as a distance to default metric. Assuming that changes in asset value are normally distributed, the default probability can be expressed as the probability of a standard normal variate falling below some critical value.

Conceptually, it is useful to anchor the default process by fixing the default threshold, for instance, at the end of the sample period, thereby allowing the loss distribution to shift in response to macroeconomic factors. Define $PD_{jit} = \Pr(r_{ji,t+1} < c_{ji} | \mathcal{I}_t)$ as shorthand notation for the probability of default of company j in region i at time t. Then, using (66) and (70), this default probability can be written as

$$PD_{jit} = \Phi\left(\frac{c_{ji} - \mu_{jit}}{\omega_{\xi,ji}}\right), \tag{71}$$

where $\Phi(\cdot)$ is the standard normal distribution function. Estimates for these default probabilities can be computed from credit rating histories provided by rating agencies, such as Moody and Standard & Poor's (S&P). For each period and each rating R_t, we can estimate the default probability for each time period, $PD_{\mathcal{R}_t}$. For example, the estimated probability of default for companies rated "$\mathcal{BBB}$" in period t may be 22 basis points (bp's) ($PD_{\mathcal{BBB}_t} = 22$ bp), whereas in period t' it may rise to 37 bp ($PD_{\mathcal{BBB}_{t'}} = 37$ bp). We are then able to assign that default probability in period t for rating $\mathcal{R}$ to all firms with that rating in that period.

Therefore, two different firms with the same credit rating in period t will have the same default probability estimates. Specifically,

$$\Pr(r_{ji,t+1} < c_{ji}|\mathcal{I}_t) = PD(\mathcal{R}_{jit})$$

and, therefore,

$$c_{ji} = \mu_{jit} + \omega_{\xi,ji} DT(\mathcal{R}_{jit}), \tag{72}$$

where $DT(\mathcal{R}_{jit}) = \Phi^{-1}(PD(\mathcal{R}_{jit}))$ is the "default threshold" associated with the estimated default probability $PD(\mathcal{R}_{jit})$ and $\Phi^{-1}(\cdot)$ denotes the inverse cumulative standard normal distribution.

Suppose now that we have time series data over the sample period $t = 1, 2, \ldots, T$, and we wish to obtain an estimate of the default threshold at T to be used in computing the conditional loss distribution over the period T to $T + 1$. Averaging the relations (72) over $t = 1$ to T, we obtain

$$c_{ji} = \bar{\mu}_{ji} + \omega_{\xi,ji}\overline{DT}_{\mathcal{R}ji},$$

where

$$\bar{\mu}_{ji} = \frac{1}{T}\sum_{t=1}^{T}\mu_{jit} \qquad \text{and} \qquad \overline{DT}_{\mathcal{R}ji} = \frac{1}{T}\sum_{t=1}^{T} DT(\mathcal{R}_{jit}).$$

A model-free estimate of $\bar{\mu}_{ji}$ is given by $\bar{r}_{ji}$, the average return over the sample period, and $\omega_{\xi,ji}$ can be estimated (as shown later) using the GVAR model and the parameters of the APT regressions. Alternatively, an unconditional (model-free) estimate of the return variance, say $\omega_{ji}^2 = \text{var}(r_{ji,t+1})$, could be used. The results are unlikely to be much affected by which of the two estimated error variances is used. But the model-free estimate has the advantage of being simple and could fit better with the rating agencies' own approach of not putting too much weight on the business cycle factors in arriving at their credit ratings. We use rating histories from Moody's to estimate $PD_{\mathcal{R}_t}$ and hence $\hat{c}_{ji}$, because "Moody's believes that giving only a modest weight to cyclical conditions best serves the interests of the bulk of investors" (Moody's Investors Services 1999, pp. 6–7).

Adopting the model-free estimation approach, c_{ji} can be consistently estimated at time T by

$$\hat{c}_{ji} = \bar{r}_{ji} + \hat{\omega}_{ji}\overline{DT}_{\mathcal{R}ji}, \tag{73}$$

where

$$\hat{\omega}_{ji}^2 = \frac{\sum_{t=1}^{T}(r_{jit} - \bar{r}_{ji})^2}{T - 1}.$$

We would say that conditional on information that we have at time T, default occurs when $r_{ji,T+1} < \hat{c}_{ji}$ or, equivalently, if

$$r_{ji,T+1} < \bar{r}_{ji} + \hat{\omega}_{ji}\overline{DT}_{\mathcal{R}ji}. \tag{74}$$

By treating the critical value as constant, we implicitly assume constant liability growth. Thus we continue to make assumptions about the capital structure of the firm, but ones that are less restrictive and more realistic.

In the Merton default prediction model, accounting data (i.e., book value of callable liabilities), the market value of equity and the volatility in the market value of equity are used to derive $PD(\mathcal{R}_{jit})$. We do the inverse; using an existing measure of expected default probability, we determine the critical value $\hat{c}_{ji}$.

There are several reasons to believe that this approach is less than ideal. Putting aside issues of the structural Merton model per se (e.g., the assumption that the value of liabilities remains unaltered even if the market value of assets may double), mappings from credit ratings to default probabilities are typically obtained using corporate bond rating histories over many years. The reason for this is simple: Default events for investment grade firms are quite rare, less than .5% per year. However, there is substantial evidence that default rates are tied to the business cycle (Nickell, Perraudin, and Varotto 2000; Bangia et al. 2002). The difficult task of endogenizing the default threshold is a fruitful area for future research.

10.2 Expected Loss Due to Default

Given the value change process for firm j, defined by (69), and the default threshold, $\hat{c}_{ji}$, we now consider the conditions under which the firm goes bankrupt and is thus no longer able to repay its debt obligations. Specifically, we need to define the expected loss to firm j at time T given information available to the lender (e.g., a bank) at time T, which we denote by $\mathcal{I}_T$. Default occurs when the firm's value (return) falls below some threshold $\hat{c}_{ji}$ (e.g., when the value of a firm's assets falls below the value of its callable liabilities). Expected loss at time T, $E_T(L_{ji,T+1}) = E(L_{ji,T+1}|\mathcal{I}_T)$, is given by

$$E_T(L_{ji,T+1}) = \Pr(r_{ji,T+1} < \hat{c}_{ji}|\mathcal{I}_T)E_T(\mathcal{X}_{ji,T+1})E_T(\mathcal{S}_{ji,T+1}) + [1 - \Pr(r_{ij,T+1} < \hat{c}_{ji}|\mathcal{I}_T)] \times \tilde{L}, \tag{75}$$

where $\hat{c}_{ji}$ is given by (73), $\mathcal{X}_{ji,T+1}$ is the maximum loss exposure assuming no recoveries for company j in region i (typically the face value of the loan) and is known at time T, $\mathcal{S}_{ji,T+1}$ is the percentage of exposure that cannot be recovered in the event of default, and $\tilde{L}$ is some future loss in the event of nondefault at $T + 1$ (which we set to zero for simplicity). Typically, $\mathcal{S}_{ji,T+1}$ is not known at time of default and will be treated as a random variable over the range [0, 1]. In the empirical application we assume that $\mathcal{S}_{ji,T+1}$ are draws from a beta distribution with given mean and variance calibrated to (pooled) historical data on default severity. Substituting (69) into (75) and setting $\tilde{L}$ to zero, we now obtain

$$E_T(L_{ji,T+1}) = \Pr\big(\alpha_{ji} + \boldsymbol{\beta}'_{ji}\Delta\mathbf{x}_{i,T+1} + \boldsymbol{\gamma}'_{ji}\Delta\mathbf{d}_{T+1} + \eta_{ji,T+1} < \hat{c}_{ji}|\mathcal{I}_T\big) \times E_t(\mathcal{X}_{ji,T+1})E_t(\mathcal{S}_{ji,T+1}). \tag{76}$$

To compute the *conditional* default probability,

$$\pi_{ji,T} = \Pr\big(\alpha_{ji} + \boldsymbol{\beta}'_{ji}\Delta\mathbf{x}_{i,T+1} + \boldsymbol{\gamma}'_{ji}\Delta\mathbf{d}_{T+1} + \eta_{ji,T+1} < \hat{c}_{ji}|\mathcal{I}_T\big), \tag{77}$$

we make use of the solution to the GVAR model given by (24) and (25) and note that

$$\Delta\mathbf{x}_{i,T+1} = \mathbf{S}_i\big[\mathbf{b}_0 + \mathbf{b}_1(T+1) - (\mathbf{I}_k - \boldsymbol{F})\mathbf{x}_T + \boldsymbol{\Upsilon}_0\Delta\mathbf{d}_{T+1} + (\boldsymbol{\Upsilon}_0 + \boldsymbol{\Upsilon}_1)\mathbf{d}_T + \mathbf{G}^{-1}\boldsymbol{\varepsilon}_{T+1}\big],$$

where $\mathbf{S}_i$ is a $k_i \times k$ selection matrix such that $\mathbf{x}_{it} = \mathbf{S}_i\mathbf{x}_T$. In the case where the macroeconomic variables are stacked by countries, as in $\mathbf{x}_T = (\mathbf{x}'_{0T}, \mathbf{x}'_{1T}, \dots, \mathbf{x}'_{NT})'$, we have $\mathbf{S}_i = (\mathbf{0}_{k_0}, \dots, \mathbf{0}_{k_{i-1}}, \mathbf{I}_{k_i}, \mathbf{0}_{k_{i+1}}, \dots, \mathbf{0}_{k_N})$. To take into account the uncertainty associated with the global exogenous variables, $\mathbf{d}_{T+1}$, we adopt the autoregressive specification defined by (34) and note that

$$\Delta\mathbf{d}_{T+1} = \boldsymbol{\mu}_d - (\mathbf{I}_s - \boldsymbol{\Phi}_d)\mathbf{d}_T + \boldsymbol{\varepsilon}_{d,T+1}. \tag{78}$$

Hence

$$\Delta\mathbf{x}_{i,T+1} = \mathbf{S}_i\big[\mathbf{b}_0 + \boldsymbol{\Upsilon}_0\boldsymbol{\mu}_d + \mathbf{b}_1(T+1) - (\mathbf{I}_s - \boldsymbol{F})\mathbf{x}_T + (\boldsymbol{\Upsilon}_0\boldsymbol{\Phi}_d + \boldsymbol{\Upsilon}_1)\mathbf{d}_T + \boldsymbol{\Upsilon}_0\boldsymbol{\varepsilon}_{d,T+1} + \mathbf{G}^{-1}\boldsymbol{\varepsilon}_{T+1}\big], \tag{79}$$

where $\boldsymbol{\varepsilon}_{d,T+1} \backsim \text{iid}(\mathbf{0}, \boldsymbol{\Sigma}_d)$ and, by assumption, is distributed independently of the macroeconomic shocks, $\boldsymbol{\varepsilon}_{T+1}$, and the firm's idiosyncratic shock, $\eta_{ji,T+1}$. Using this result in (77), and after some simplifications, we have

$$\pi_{ji,T} = \Pr(\xi_{ji,T+1} < \hat{c}_{ji} - \mu_{ji,T}|\mathcal{I}_T), \tag{80}$$

where

$$\xi_{ji,T+1} = \eta_{ji,T+1} + \boldsymbol{\theta}'_{ji}\boldsymbol{\varepsilon}_{T+1} + \boldsymbol{\theta}'_{ji,d}\boldsymbol{\varepsilon}_{d,T+1}, \tag{81}$$

$$\boldsymbol{\theta}'_{ji} = \boldsymbol{\beta}'_{ji}\mathbf{S}_i\mathbf{G}^{-1}, \qquad \boldsymbol{\theta}'_{ji,d} = \boldsymbol{\gamma}'_{ji} + \boldsymbol{\beta}'_{ji}\mathbf{S}_i\boldsymbol{\Upsilon}_0, \tag{82}$$

and

$$\mu_{ji,T} = \alpha_{ji} + \boldsymbol{\gamma}'_{ji}\boldsymbol{\mu}_d + \boldsymbol{\beta}'_{ji}\mathbf{S}_i(\mathbf{b}_0 + \mathbf{b}_1 + \boldsymbol{\Upsilon}_0\boldsymbol{\mu}_d) + \boldsymbol{\beta}'_{ji}\mathbf{S}_i[\mathbf{b}_1 T - (\mathbf{I}_k - \boldsymbol{F})\mathbf{x}_T] + [\boldsymbol{\beta}'_{ji}\mathbf{S}_i(\boldsymbol{\Upsilon}_0\boldsymbol{\Phi}_d + \boldsymbol{\Upsilon}_1) - \boldsymbol{\gamma}'_{ji}(\mathbf{I}_s - \boldsymbol{\Phi}_d)]\mathbf{d}_T. \tag{83}$$

Therefore, there are three types of shocks that affect a firm's probability of default: its own shock, $\eta_{ji,T+1}$; macroeconomic shocks, $\boldsymbol{\varepsilon}_{T+1}$; and the global exogenous shock, $\boldsymbol{\varepsilon}_{d,T+1}$ (in our model, the oil price shock). Note that although the firm in question operates in country/region i, its probability of default could be affected by macroeconomic shocks worldwide. Under the assumption that all of these shocks are jointly normally distributed and the parameter values are given, we have the following expression for the probability of default over T to $T+1$ formed at T:

$$\pi_{ji,T} = \Phi\left[\frac{\hat{c}_{ji} - \mu_{ji,T}}{\sqrt{\text{var}(\xi_{ji,T+1}|\mathcal{I}_T)}}\right], \tag{84}$$

where

$$\text{var}(\xi_{ji,T+1}|\mathcal{I}_T) \equiv \omega^2_{\xi,ji} = \omega^2_{\eta,ji} + \boldsymbol{\theta}'_{ji}\boldsymbol{\Sigma}\boldsymbol{\theta}_{ji} + \boldsymbol{\theta}'_{ji,d}\boldsymbol{\Sigma}_d\boldsymbol{\theta}_{ji,d}. \tag{85}$$

Both of these restrictions (given parameter values and joint normality) can be relaxed. Parameter uncertainty can be taken into account by integrating out the unknown parameters using their posterior or predictive likelihoods, as was done by Garratt, Lee, Pesaran, and Shin (2003b). In the presence of nonnormal shocks, one could also use nonparametric stochastic simulation techniques by resampling from the residuals of the GVAR model to estimate $\pi_{ji,T}$. These and other related developments are beyond the scope of the present application, which is intended primarily as an illustration of the GVAR modeling approach in credit risk analysis.

The expected loss due to default of a loan (credit) portfolio can now be computed by aggregating the expected losses across the different loans. Denoting the loss of a loan portfolio over the period T to $T+1$ by L_{T+1}, we have

$$E_T(L_{T+1}) = \sum_{i=0}^{N}\sum_{j=1}^{nc_i} \pi_{ji,T}E_T(\mathcal{X}_{ji,T+1})E_T(\mathcal{S}_{ji,T+1}), \tag{86}$$

where nc_i is the number of obligors (which could be 0) in the bank's loan portfolio resident in country/region i.

10.3 Simulation of the Loss Distribution

The expected loss, as well as the loss distribution, can also be computed by stochastic simulation using draws from the joint distribution of the shocks, $\boldsymbol{\epsilon}_{T+1} = (\boldsymbol{\eta}'_{T+1}, \boldsymbol{\varepsilon}'_{T+1}, \boldsymbol{\varepsilon}'_{d,T+1})'$, where $\boldsymbol{\eta}_{T+1}$ is the vector of firm-specific shocks. As noted earlier, these draws could either be carried out parametrically from normal or t-distributed random variables or, if sufficient data points are available, implemented nonparametrically using resampling techniques. Under the parametric specification, the variance-covariance matrix of $\boldsymbol{\epsilon}_{T+1}$ is given by

$$\text{cov}(\boldsymbol{\epsilon}_{T+1}) = \begin{pmatrix} \boldsymbol{\Sigma} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_d & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\Theta} \end{pmatrix}, \tag{87}$$

where $\boldsymbol{\Theta}$ is a diagonal matrix with elements ω^2_{ji}, $j = 1, 2, \dots, nc_i$, $i = 0, 1, \dots, N$.

Denote the rth draw of this vector by $\boldsymbol{\epsilon}^{(r)}_{T+1}$, and compute the firm-specific return, $r^{(r)}_{ij,T+1}$, noting that

$$r^{(r)}_{ij,T+1} = \mu_{ji,T} + \xi^{(r)}_{ji,T+1}, \tag{88}$$

where $\mu_{ji,T}$ is given by (83) and

$$\xi^{(r)}_{ji,T+1} = \eta^{(r)}_{ji,T+1} + \boldsymbol{\theta}'_{ji}\boldsymbol{\varepsilon}^{(r)}_{T+1} + \boldsymbol{\theta}'_{ji,d}\boldsymbol{\varepsilon}^{(r)}_{d,T+1}. \tag{89}$$

Then simulate the loss in period $T+1$ using (known) loan face values, say $FV_{ji,T}$, as exposures, and draws from a beta distribution for severities (as described earlier),

$$L^{(r)}_{T+1} = \sum_{i=0}^{N}\sum_{j=1}^{nc_i} I\big(r^{(r)}_{ij,T+1} < \hat{c}_{ji}\big)FV_{ji,T}\mathcal{S}^{(r)}_{ji,T+1}. \tag{90}$$

The simulated expected loss due to default is given by (using R replications)

$$\bar{L}_{R,T+1} = \frac{1}{R}\sum_{r=1}^{R} L_{T+1}^{(r)}. \qquad (91)$$

When $\boldsymbol{\epsilon}_{T+1}^{(r)}$ are drawn from a multivariate normal distribution with a covariance matrix given by (87), we have

$$\bar{L}_{R,T+1} \xrightarrow{p} E_T(L_{T+1}) \qquad \text{as } R \to \infty.$$

The simulated loss distribution is given by ordered values of $L_{T+1}^{(r)}$, for $r = 1, 2, \ldots, R$. For a desired percentile, for example 99%, and a given number of replications, say $R = 10{,}000$, credit value at risk is given as the 100th highest loss.

10.4 Expected Loss Given Shocks

In credit risk analysis, we may also be interested in evaluating quantitatively the relative importance of changes in different macroeconomic factors on the loss distribution. To this end, the loss distribution conditional on a given shock can be compared with a baseline distribution without such a shock. As with all counterfactual experiments, it is important that the effects of the shock on other macroeconomic factors be clearly specified. One possibility would be to assume that the other factors are displaced according to their historical covariances with the variable being shocked. This is in line with the GIRF analysis discussed earlier. In this setup, if factor ℓ in country i is shocked by one standard error (i.e., $\sqrt{\sigma_{ii,\ell\ell}}$) in the period from T to $T+1$, then the vector of the macroeconomic factors would be displaced by

$$\frac{1}{\sqrt{\sigma_{ii,\ell\ell}}}\mathbf{G}^{-1}\boldsymbol{\Sigma}\boldsymbol{\mathfrak{s}}_{\ell},$$

where, as before, $\boldsymbol{\mathfrak{s}}_{\ell}$ is a $k \times 1$ selection vector with its element corresponding to the ℓth variable being unity in country i and zero elsewhere. Note also that $\varepsilon_{i,T+1,\ell} = \boldsymbol{\mathfrak{s}}_{\ell}'\boldsymbol{\varepsilon}_{T+1} = \sqrt{\sigma_{ii,\ell\ell}}$. Such a shock has no effect on the global exogenous variables and the firm-specific shocks. In the absence of any macroeconomic shocks, namely when $\boldsymbol{\varepsilon}_{T+1} = \mathbf{0}$, firm-specific returns are given by

$$r_{ij,T+1}^{0} = \mu_{ji,T} + \eta_{ji,T+1} + \boldsymbol{\theta}_{ji,d}'\boldsymbol{\varepsilon}_{d,T+1},$$

and with a one standard error shock to $x_{i,T+1,\ell}$ (or, more specifically, conditional on $\varepsilon_{i,T+1,\ell} = \boldsymbol{\mathfrak{s}}_{\ell}'\boldsymbol{\varepsilon}_{T+1} = \sqrt{\sigma_{ii,\ell\ell}}$), we have [see (87) and (88)]

$$r_{ij,T+1}^{\ell} = \mu_{ji,T} + \eta_{ji,T+1} + \frac{1}{\sqrt{\sigma_{ii,\ell\ell}}}\boldsymbol{\theta}_{ji}'\boldsymbol{\Sigma}\boldsymbol{\mathfrak{s}}_{\ell} + \boldsymbol{\theta}_{ji,d}'\boldsymbol{\varepsilon}_{d,T+1}.$$

The loss distributions associated with these two scenarios can now be simulated using these returns in (90).

The foregoing counterfactual, although of some interest, will underestimate the expected loss under both shock scenarios, because it abstracts from volatility of the macroeconomic factors. To allow for the volatility of macroeconomic factors in the analysis, one could distinguish between the case where the shock (or intervention) is preannounced or is anticipated and the case where the shock is unanticipated. Here we focus on the latter case, which is arguably more relevant to risk analysis. We also assume that the magnitude and the nature of the shock are not such as to alter the shape of the probability distribution function of $\boldsymbol{\varepsilon}_t$ (for further discussion, see PSTW).

Assuming that $\boldsymbol{\theta}_{ji}'\boldsymbol{\varepsilon}_{T+1}$ and $\varepsilon_{i,T+1,\ell} = \boldsymbol{\mathfrak{s}}_{\ell}'\boldsymbol{\varepsilon}_{T+1}$ are jointly normally distributed and the shock is unanticipated, it is then easily seen that

$$\boldsymbol{\theta}_{ji}'\boldsymbol{\varepsilon}_{T+1}|\varepsilon_{i,T+1,\ell} = \sqrt{\sigma_{ii,\ell\ell}} \sim \text{IIN}\left(\frac{1}{\sqrt{\sigma_{ii,\ell\ell}}}\boldsymbol{\theta}_{ji}'\boldsymbol{\Sigma}\boldsymbol{\mathfrak{s}}_{\ell}, \boldsymbol{\theta}_{ji}'\boldsymbol{\Sigma}\boldsymbol{\theta}_{ji}\right).$$

Also, recalling that $\boldsymbol{\varepsilon}_{T+1}$, $\boldsymbol{\varepsilon}_{d,T+1}$, and $\boldsymbol{\eta}_{T+1}$ are independently distributed, it is then easily seen that

$$\xi_{ij,T+1}|\varepsilon_{i,T+1,\ell} = \sqrt{\sigma_{ii,\ell\ell}} \sim \text{IIN}\left(\frac{1}{\sqrt{\sigma_{ii,\ell\ell}}}\boldsymbol{\theta}_{ji}'\boldsymbol{\Sigma}\boldsymbol{\mathfrak{s}}_{\ell}, \omega_{\xi,ji}^{2}\right), \qquad (92)$$

where $\omega_{\xi,ji}^{2}$ is as defined by (85).

Therefore, to allow for volatility of the shocks (both macroeconomic and idiosyncratic shocks), the simulation of the loss distribution needs to be carried out using the draws

$$r_{ij,T+1}^{l,(r)} = \mu_{ji,T} + \frac{1}{\sqrt{\sigma_{ii,\ell\ell}}}\boldsymbol{\theta}_{ji}'\boldsymbol{\Sigma}\boldsymbol{\mathfrak{s}}_{\ell} + \omega_{\xi,ji}\mathcal{Z}^{(r)}, \qquad (93)$$

where $\mathcal{Z}^{(r)} \sim \text{IIN}(0, 1)$. The baseline loss distribution in this case can also be simulated directly using the draws

$$r_{ij,T+1}^{(r)} = \mu_{ji,T} + \omega_{\xi,ji}\mathcal{Z}^{(r)}, \qquad (94)$$

Default occurs if the rth simulated return, baseline ($r_{ij,T+1}^{(r)}$) or shock-conditional ($r_{ij,T+1}^{l,(r)}$), falls below the threshold $\hat{c}_{ji}$:

$$\text{Baseline:} \quad r_{ij,T+1}^{(r)} < \hat{c}_{ji} \implies \text{default}, \qquad (95)$$

$$\text{shock-conditional:} \quad r_{ij,T+1}^{l,(r)} < \hat{c}_{ji} \implies \text{default}.$$

Using these results in (90), the loss distribution can be simulated for any desired level of accuracy by selecting R, the number of replications, to be sufficiently large.

10.5 Results

10.5.1 The Sample Portfolio. We analyze the effects of economic shocks on a hypothetical sample of a large-corporate loan portfolio comprised of 119 companies, dispersed over 10 regions, with a current face value of \$1 billion, the same portfolio used in PSTW. We restricted ourselves to major, publicly traded firms that had a credit rating from either Moody's or S&P. Thus, for example, Chinese companies are not included for lack of a credit rating. Further details are provided in PSTW. Table 13 provides the individual company details, with a summary by regions given in Table 14 (which is table 4 in PSTW). The column to the right indicates the inception of the equity series available for APT-type regression analysis. We wanted to mimic (broadly) the portfolio of a large, internationally active bank. Arbitrarily picking Germany as the bank's domicile country, the portfolio is relatively more exposed to German firms than would be the case were exposure allocated purely on a GDP share (in our "world" of 25 countries). For the remaining regions, exposure was more in line with GDP share. Within a region, loan exposure is assigned randomly. The expected severity for loans to U.S. companies is the lowest at 20%, based on studies by Citibank, Fitch Investor Service, and Moody's Investors Service. All other severities are based on assumptions, reflecting the idea that severities are higher in less-developed countries.

Table 13. Loan Portfolio Details

Country	*Company*	*Exposure ($)*	*Expected severity (%)*	*Unexpected severity (%)*	*Credit rating*
U.S.	General Motors	24,403,425.67	20	10	A
	Procter & Gamble	7,642,331.88	20	10	AA
	Du Pont E I De Nemours	8,804,944.35	20	10	AA−
	Merrill Lynch	5,742,903.17	20	10	AA−
	Merck	13,888,942.68	20	10	AAA
	Alcoa	9,258,999.32	20	10	A+
	Intl. Paper	15,245,642.52	20	10	BBB
	Minnesota Mng. & Mnfg.	13,760,015.63	20	10	AA
	Dow Chemicals	18,667,507.97	20	10	A
	Eastman Kodak	9,296,388.28	20	10	A+
	Exxon Mobil	26,461,015.08	20	10	AAA
	Dole Food	13,267,258.80	20	10	BBB−
	General Electric	14,720,578.43	20	10	AAA
	AT&T	18,840,046.23	20	10	A
U.K.	Unilever (UK)	4,276,017.85	35	15	A+
	Barclays	20,019,910.39	35	15	AA
	Prudential Corp.	9,105,506.84	35	15	AA
	HSBC Bank	18,434,016.27	35	15	A
	Hanson	1,175,919.51	35	15	BBB+
	EMI Group	5,739,190.67	35	15	BBB+
	British Petroleum	2,883,742.89	35	15	AA+
	Cadbury Schweppes	2,856,419.56	35	15	A
	GlaxoSmithKline	15,509,276.03	35	15	AA
Germany	Allianz	4,907,699.72	30	15	AAA
	Basf	12,736,292.64	30	15	AA−
	Bayer	3,610,483.11	30	15	AA
	Bayer. Hypo-Und-Vbk.	21,076,958.20	30	15	A+
	BMW	25,204,668.59	30	15	A1
	Continental	21,314,020.03	30	15	BBB+
	Deutsche Bank	5,681,228.25	30	15	AA
	Dresdner Bank	11,065,314.60	30	15	AA−
	Dyckerhoff Pref.	1,644,590.99	30	15	BBB
	Eon	595,090.57	30	15	AA
	Ergo Versicherung	18,316,378.67	30	15	AA+
	Heidelb. Zement	545,743.55	30	15	BBB+
	Linde	13,318,432.99	30	15	A−
	MG Technologies	15,628,726.49	30	15	Baa3
	RWE	10,513,827.39	30	15	AA−
	Siemens	12,984,846.92	30	15	AA
	Volkswagen	21,221,887.10	30	15	A+
	Mannesmann	9,633,810.19	30	15	A2
France	Danone	10,808,444.66	35	15	A+
	Accor	21,822,153.43	35	15	BBB
	Axa	11,578,198.89	35	15	A+
	Peugeot Sa	3,932,011.47	35	15	A−
	Societe Generale	10,327,239.20	35	15	AA−
	Carrefour	5,920,417.33	35	15	AA
	Total Fina Elf	8,069,209.25	35	15	BBB
	Alcatel	7,542,325.78	35	15	A+
Italy	Banca Di Roma	15,102,669.83	35	15	A2
	Olivetti	488,289.81	35	15	BBB
	Parmalat	16,971,121.98	35	15	BBB−
	Assicurazioni Generali Spa	3,103,777.83	35	15	AA
	Telecom Italia	19,657,983.85	35	15	Baa1
	Banca Nazionale Del Lavoro Spa	24,676,156.70	35	15	BBB
Spain	Banco Santander Central Hispano S.A.	9,027,487.61	35	15	A+
	Telefonica	5,901,399.11	35	15	A+
	Repsol-Ypf	3,658,998.46	35	15	BBB+
Netherlands	Royal Dutch Ptl.	3,779,777.46	35	15	AAA
	Elsevier	2,521,625.71	35	15	A+
	Unilever	8,788,495.47	35	15	A+
	Aegon	9,179,570.94	35	15	AA−
	Koninklijke Philips Electronic	10,209,157.00	35	15	BBB+
Switzerland	Novartis	886,420.83	35	15	AAA
	ABB Ltd.	7,472,191.10	35	15	AA−
	Nestle	10,300,307.11	35	15	AAA
Belgium	Solvay	8,274,569.20	35	15	A2
Turkey	Turkiye Is Bank	4,433,667.94	60	20	B−
	Garanti Bankasi	8,455,811.73	60	20	B−
	Yapi Kredi Bank	863,452.99	60	20	B−
	Akbank	6,247,067.35	60	20	B−

Table 13. (continued)

Country	Company	Exposure ($)	Expected severity (%)	Unexpected severity (%)	Credit rating
Korea	Korea Electric Power	4,925,144.42	50	20	BBB
	Korea Development	2,248,774.21	50	20	Baa2
	Hyundai Motor	5,547,629.00	50	20	BB
	Seoul Bank	2,807,680.36	50	20	B
	Kumgang Korea Chemical	2,608,699.81	50	20	BBB−
	Samsung Electronics	7,335,028.98	50	20	BBB−
	Pohang Iron Steel (Posco)	5,340,221.26	50	20	BBB
	Korea First Bank	3,476,969.41	50	20	BB+
Malaysia	Southern Bank	8,086,349.82	50	20	BB
	Public Bank	3,152,851.78	50	20	BBB
	Malayan Banking	8,162,898.12	50	20	BBB−
Philippines	Philp. Long Dsn. Tel.	2,850,560.19	50	20	BB+
Singapore	United Overseas Bank	3,964,152.09	50	20	A
	Overseas Union Bank	363,382.29	50	20	BBB
	Overseas Chinese Bkg.	2,603,048.75	50	20	A
Thailand	Thai Farmers Bank	4,306,832.75	50	20	BB
	Siam Commercial Bank	7,691,370.22	50	20	B+
	Siam City Bank	6,906,233.19	50	20	B
	Thai Military Bank	555,493.82	50	20	B
	Indl. Fin. Corp. of Thai.	377,297.16	50	20	BBB−
	Bangkok Bank	8,083,616.23	50	20	BB
	Bank of Asia	2,541,786.31	50	20	BB
	Bank of Ayudhya	6,063,979.82	50	20	B+
Japan	Toyota Motor	1,577,113.58	35	15	AAA
	Nissan Motor	11,788,318.46	35	15	BB+
	Daiwa Bank	10,222,900.12	35	15	BB+
	Sumitomo Mitsu Bkg.	4,868,604.22	35	15	BBB+
	Konica	13,302,520.22	35	15	Baa2
	Toshiba	13,729,875.91	35	15	BBB+
	Sony	6,693,193.60	35	15	A+
	Hitachi	2,539,134.57	35	15	A+
	Mitsui Engr. & Shipbldg.	7,538,811.87	35	15	B1
	Sapporo Breweries	10,529,179.07	35	15	Ba3
	Asahi Glass Co. Ltd.	5,930,380.07	35	15	A2
	Japan Airlines	365,098.43	35	15	Baa3
	Makita Corp.	10,914,869.89	35	15	A2
Chile	Gener	4,115,317.41	65	20	BBB
	Vapores	3,445,260.18	65	20	BBB
	Enersis	4,865,410.24	65	20	A
	Entel	6,338,152.32	65	20	BBB−
	Chilectra	4,909,541.11	65	20	A−
Argentina	Pecom Enga.	567,141.29	65	20	BB
	Acindar	5,725,702.05	65	20	B−
Mexico	Apasco	1,986,266.98	65	20	BBB+
	Cemex	4,955,280.49	65	20	BBB−
	Desc	5,614,317.80	65	20	BBB−
	Vitro	5,407,823.19	65	20	BB
Brazil	Petrobras	2,069,786.94	65	20	B2

Table 14. Composition of the Sample Portfolio for Regions

Region	No. of obligors	Equity series[a] quarterly	Credit rating[b] range	Portfolio percentage	Severity[c] Mean (μ_β)	SD (σ_β)
U.S.	14	79Q1–99Q1	AAA to BBB−	20	20%	10%
U.K.	9	79Q1–99Q1	AA to BBB+	6	35%	15%
Germany	18	79Q1–99Q1	AAA to BBB−	21	30%	15%
France	8	79Q1–99Q1	AA to BBB	8	35%	15%
Italy	6	79Q1–99Q1	A to BBB−	8	35%	15%
West Europe	12	79Q1–99Q1	AAA to BBB+	8	35%	15%
Middle East	4	90Q3–99Q1	B−	2	60%	20%
Southeast Asia	23	89Q3–99Q1	A to B	10	50%	20%
Japan	13	79Q1–99Q1	AAA to B+	10	35%	15%
Latin America	12	89Q3–99Q1	A to B−	5	65%	20%
Total	119			100		

[a] Equity prices of companies in emerging markets are not available over the full sample period used for the estimation horizon of the GVAR.
[b] The sample contains a mix of Moody's and S&P ratings, although S&P rating nomenclature is used for convenience.
[c] Severity is drawn from a beta distribution with mean μ_β and standard deviation σ_β.

10.6 Conditional Loss Distributions

The systematic risk in our model is captured empirically through the APT regressions, where firm returns are regressed on changes in all domestic variables and oil prices. In PSTW, a more thorough model selection process was considered in which all variables, foreign and domestic, are eligible. Around 80% of those regressions were significant (using the F-test) at the 5% level, with real equity prices being the most statistically important regressor, followed by the oil price and the real exchange rate variables.

We then generated loss distributions for two different horizons: one quarter ahead and four quarters ahead. A 1-year horizon is typical for credit risk management and thus of particular interest. For each horizon, we examined the impact of several shock scenarios, including those presented in Section 9. Note that 2.33σ corresponds, in the Gaussian case, to the 99% value at risk, a typical range in risk management. The scenarios are as follows:

- A -2.33σ shock to U.S. equity, corresponding to a quarterly drop of 14.28%
- A $+2.33\sigma$ shock to real German output, corresponding to a quarterly rise of 2.17%
- A -2.33σ shock to Southeast Asian equity, corresponding to a quarterly drop of 24.77%.

In addition, we present a symmetric positive shock to Southeast Asian equity prices, but found that the impact on losses was not symmetric.

We generated 50,000 simulations for each case. To ensure convergence, we also performed simulations up to 200,000 runs; the results were indistinguishable. For the one-quarter-ahead forecast and shock scenarios, we computed expected loss results, both theoretical [using (86)] and simulated [using (91)]. The two sets of estimates turn out to be very close indeed, and so we report only the simulated ones. The simulated expected loss results together with the unexpected counterparts (SD) are summarized in Table 15.

The U.S. equity price shock seems rather severe at first; expected loss is nearly double than what is expected under the baseline (no shock) scenario, whereas unexpected loss (i.e., the loss standard deviation) is about one-third higher. At the tail (99% and beyond) of the loss distribution (Fig. 8), the absolute differences are less pronounced.

For the baseline, there is a 1% chance of losing about 41.5 bp of the face value of the portfolio after one quarter, whereas conditional on the -2.33σ U.S. real equity price shock, the loss is closer to 58.2 bp. The two loss scenarios diverge further out in the tail such that at the 99.7% level, there are losses of about 53.3 bp for the baseline scenario and 75.9 bp for the U.S. equity price shock scenario. This nonlinearity is a direct consequence of the nonlinearity of the credit risk model, which can be uncovered in the loss distribution through simulation. The positive German output shock has little bearing on the loss distribution in terms of expected and unexpected loss or even on the shape of the loss distribution itself. In fact, the positive shock to Southeast Asian real equity prices is more beneficial. Thus, from the perspective of a German risk manager, the perspective that we are trying to mimic, given this portfolio, positive shocks to German output are less cause for excitement than positive shocks to Southeast Asian equity prices.

Table 15. Simulated Mean and Standard Deviation of Losses for One Quarter and Four Quarters Ahead (in basis points exposure)

Shock scenarios	*One-quarter ahead* Mean	SD	*Four-quarters ahead* Mean	SD
-2.33σ U.S. equity	3.5	12.0	6.8	31.3
-2.33σ Southeast Asian equity	2.3	9.4	5.5	28.6
Baseline	1.2	6.7	4.0	24.5
$+2.33\sigma$ German output	1.2	6.9	3.9	24.2
$+2.33\sigma$ Southeast Asian equity	1.0	6.3	3.5	22.9

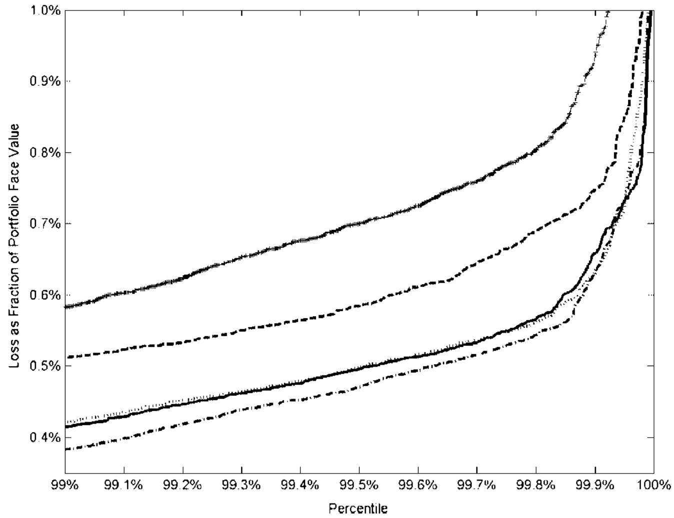

Figure 8. Shock Impacts on Credit Portfolio Loss in 1 Quarter, 50K Replications (—+— U.S. equity, negative 2.33 SD; - - - - Southeast Asia equity, negative 2.33 SD; —— baseline; ······· Germany output, positive 2.33 SD; -·-· Southeast Asia equity, positive 2.33 SD).

Symmetric shocks do not translate to symmetric loss outcomes. The loss curve shown in Figure 8 for the negative Southeast Asian equity shock lies further above the baseline than the positive equity shock curve lies below it. This is also a result of the nonlinear credit loss model.

The four-quarter-ahead loss distribution was generated one quarter at a time sequentially, allowing for the autocorrelation of return forecasts at different horizons (see PSTW for details). Means and standard deviations of the annual simulated loss distributions are presented in Table 15; the loss distributions for the baseline and the four shock scenarios are displayed in Figure 9.

The expected loss for the U.S. equity shock scenario is now about one-third higher than the baseline at the four-quarter-ahead horizon, and the pattern of the loss curves are broadly in line with the curves for the one-quarter losses, except that the loss distributions for the favorable shocks are now relatively closer to the baseline distribution. The four-quarter-ahead loss distribution is also somewhat smoother than the one-quarter-ahead loss distribution, lacking the "elbow" in the 99.7–99.8% range.

What might be the impact on losses of a severe shock, say to U.S. equity prices? From their peak in 2000 to a recent low in early October 2002, the S&P500 dropped about 49%. That also corresponds to the largest quarterly drop in the index since

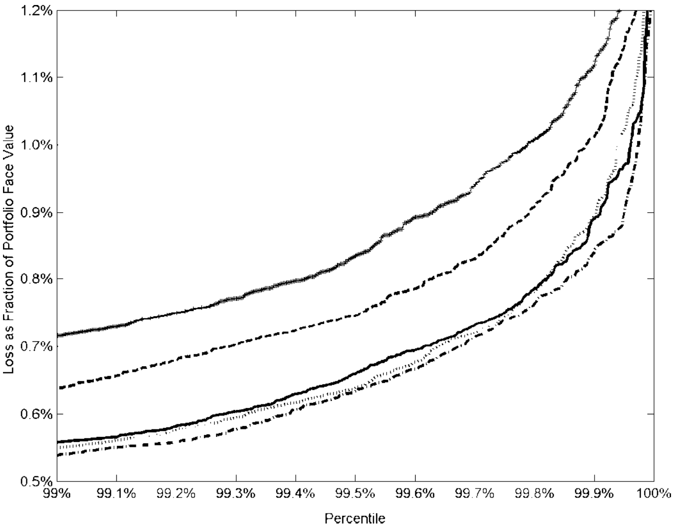

Figure 9. Shock Impacts on Credit Portfolio Loss in 4 Quarters, 50K Replications (—+— U.S. equity, negative 2.33 SD; --- Southeast Asia equity, negative 2.33 SD; — baseline; ······ Germany output, positive 2.33 SD; -·-·- Southeast Asia equity, positive 2.33 SD).

1928 (which occurred during February to April of 1932). Such a large drop corresponds to 8.02σ, and the impact on the loss distribution of our portfolio can be seen in Figure 10, which presents the one-quarter-ahead loss distribution of this stress scenario and the baseline; we also include the previous, less severe U.S. equity shock plus an intermediate shock of -5σ for comparison.

Indeed, such a shock would result in rather large losses. We would expect to lose 138.1 bp (or 1.4%) of total loan exposure, and there is a 1% chance that 3.08% of the portfolio would be wiped out. Note that total U.S. exposure in the loan book is 20%. The nonlinear impact of shocks on losses is quite pronounced: the -8.02σ shock is only 60% higher than the -5σ shock, in units of σ, of course, but the unexpected loss after one quarter is more than double (63.1 bp vs. 30.1 bp) and the 99% loss nearly three times as much (3.08% vs. 1.26%).

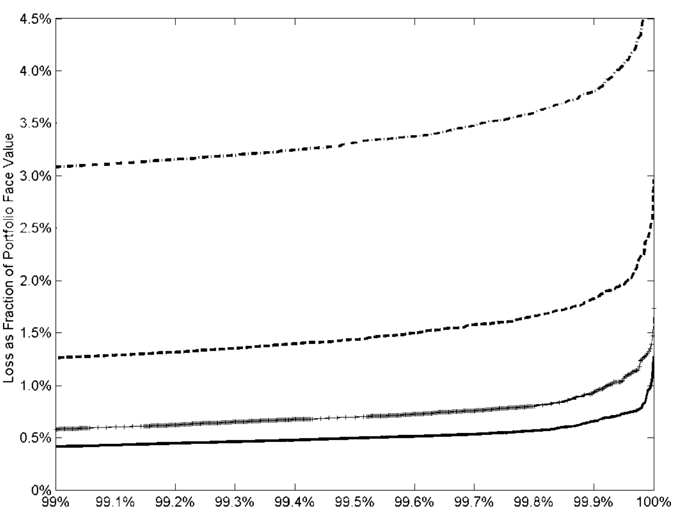

Figure 10. Shock Impacts on Credit Portfolio Loss in 1 Quarter, 50K Replications (-·-·- U.S. equity, negative 8.02 SD; --- U.S. equity, negative 5.00 SD; —+— U.S. equity, negative 2.33 SD; — baseline).

11. CONCLUDING REMARKS

In this article we have developed an operational framework for global macroeconomic modeling. Our approach aggregates regional cointegrated systems into a unified global system. We demonstrate the feasibility of this approach by linking up 11 separate vector error-correcting regional models estimated using quarterly observations over the period 1979Q1–1999Q1. Each of the regional models contains foreign variables that are weighted averages of the domestic variables for other regions, constructed to match the international trade pattern of the country under consideration. The individual country models are then combined in a consistent and cohesive manner to generate forecasts for *all* of the variables in the world economy simultaneously.

This resultant model is shown also to be error-correcting with dampened cyclical properties. We outline conditions of weak exogeneity of the foreign variables, a key assumption of the model. We then test these conditions, where we include the global variable (price of oil) in the exogeneity regressions as well. Of the 63 exogeneity tests carried out, only 3 are statistically significant at the 5% level, and none are statistically significant at the 3% level. Finally, using GIR analysis, we examine the propagation of shocks across factors and regions.

The focus of the model is very much on constructing a compact and coherent representation of factor and regional interdependencies, while tackling the problem of limited data in large-scale models such as these. Our model allows for interaction among the different economies through three separate but interrelated channels:

1. Direct dependence of the relevant macro factors on their region-specific foreign counterparts and their lagged values
2. Dependence of the region-specific variables on common global weakly exogenous variables such as oil prices and possibly other variables controlling for major global political events
3. Certain degrees of dependence of idiosyncratic shocks across regions, as captured via the cross-region covariances.

Thus, for instance, we are able to account for both long-run and short-run interlinkages between equity market movements in Southeast Asia and output in Germany. In particular, the GVAR structure, by allowing for the existence of cointegrating relations between domestic and foreign variables, avoids the criticism of Banerjee, Marcellino, and Osbat (2002) of panel cointegration techniques advanced in the literature by Kao (1999), Pesaran et al. (1999), Pedroni (2000), Breitung (2002), and Groen and Kleibergen (2003), for example, that restrict the long-run relations to depend *only* on the domestic variables.

The original motivation for developing this model was the need for a macro-based risk management tool for commercial, and perhaps even central banks. By engaging in commercial lending to companies whose fortunes fluctuate with aggregate demand, a bank is ultimately exposed to macroeconomic fluctuations. This can be mitigated through international diversification. However, precisely because economic fluctuations are correlated across factors and countries, it fosters the need for a compact global macroeconometric model that explicitly allows for such interdependencies. To demonstrate the value of constructing such a model as a basis for portfolio risk management, we use a simplified version of a Merton-style credit risk model, developed fully in PSTW, which has explicit links to the macroeconomic factors in the GVAR model, thereby allowing us to generate scenario-based loss distributions for a credit portfolio. Using a portfolio of loans to 119 firms in 10 of the 11 regions (China was left out due to poorly developed equity markets), we generated loss distributions for one quarter and four quarters ahead under both a baseline forecast and a set of shock scenarios. The simulated losses are shown to converge quickly to their analytical counterparts. We found that symmetric shocks do not result in symmetric loss outcomes due to the nonlinearity of the credit model. Our results may be thought of as demonstrating the value of hedging credit risk with market risk, an idea that is quickly gaining traction among practitioners today.

Because of the focus on modeling interlinkages, the model can be readily used to shed light on the analysis of various transmission mechanisms, contagion effects, and testing of long-run theories (e.g., PPP) in a global setting as well as other settings. Several other applications of our methodology come to mind:

- "New economic geography." A literature that sets the stage for explicitly incorporating geography into the models of economic activity through either domestic or international trade (see Krugman 1993 for an introduction to the topic, and Fujita, Krugman, and Venables 1999 for a more formal treatment)
- Regional and urban economics. Models of interregional linkages, either through city–suburb economic ties (Voith 1998) or linkages between cities, as in the "systems of cities" literature (Henderson 1988)
- Labor mobility. Consider a longer-horizon, lower-frequency issue of labor mobility responding to regional economic shocks; for instance, auto workers migrating from Michigan to Texas in response to oil price shocks in the early 1980s (Blanchard and Katz 1992).

This list is by no means exhaustive and is designed to stimulate interest in, and research into, applying the GVAR framework to problems of modeling economic interlinkages.

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APPENDIX A: DATA

A.1 Variables and Data Sources

The primary variables (disaggregated by country/region when applicable) used in this study are:

Y :	GDP
P :	General price index
Q :	Equity price index
E :	Exchange rate
R :	Interest rate
M :	Money supply
PO :	Oil price

A.2 Output (GDP)

The source for all 25 countries is the IMF's International Financial Statistics (IFS) GDP (1990) series. France, Germany, Italy, Japan, Mexico, the Netherlands, Spain, Switzerland, the U.K., and the U.S. are all from series BR, and the remaining countries are from series BP.

Where quarterly data were not available (i.e., for Brazil, China, Indonesia, Kuwait, Malaysia, Saudi Arabia, Singapore, Thailand, and Turkey), quarterly series were interpolated linearly from the annual series. Interpolated series were used only during the periods 1979–1992 for Singapore, 1979–1996 for Malaysia, and 1979–1995 for Thailand. Quarterly output series were available for the subsequent periods.

For the period before German reunification, in 1990Q4, West German growth rates were used. The growth rate from 1988Q3 to 1990Q3 was used to compute a "unified" output series for 1990Q4.

The data for Kuwait and Peru were rebased to 1990 using the CPI for those countries. The data for Argentina and Singapore were seasonally adjusted.

A.3 General Price Indices

The data source for all countries except China was the IFS Consumer Price Index Series 64. A full sample was available for all countries except Brazil, where 1979 data were unavailable and a backcast using the average growth rate of prices for 1980 was used.

A.4 Equity Price Indices

There were no data available for China or Saudi Arabia. For Belgium, Indonesia, Italy, Malaysia, Singapore, Spain, Switzerland, Thailand, and Turkey we used Datastream, using quarterly averages from daily observations. However, we used quarterly averages of weekly datapoints, as opposed to daily observations, for Argentina. The data for Malaysia were market cap-weighted.

We used IFS data for Brazil, Chile, France, Germany, Japan, Korea, Mexico, the Netherlands, Peru, Philippines, the U.K., and the U.S. Indices for share prices (IFS code 62) generally related to common shares of companies traded on national or foreign stock exchanges. Monthly indices were obtained as simple arithmetic averages of the daily or weekly observations (ZF).

These nominal equity price indices were deflated by the nonseasonally adjusted general price indices. The resultant real series were then adjusted for (possible) seasonal variations.

A.5 Exchange Rates

IFS series rf was used for all countries.

A.6 Interest Rates

Interest rate data were taken from IFS Series 60B, the money market rate, with the following exceptions: For Argentina, Chile, China, Saudi Arabia, and Turkey, we used the IFS deposit rate; for Peru, we used the IFS discount rate; and for the Philippines, we used the IFS Treasury rate.

A.7 Money Supply

The money supply data source for all countries was the sum of IFS series 34 (money) and series 35 (quasi-money). All series were seasonally adjusted. The data for Argentina, Brazil, Peru, and Turkey required a decimal place adjustment to make the money:GDP ratio reasonable.

For Belgium, we used quarterly data for all quasi-money; for money, we used annual data converted to quarterly through interpolation up to 1990, and quarterly data from 1990Q4 to 1999Q1.

We used annual data converted to quarterly through interpolation for the Philippines; for the Philippines, this was necessary for the period 1984–1986 only, because quarterly data were available thereafter. There were no quarterly data available for Saudi Arabia for 1983, and thus we used annual data for that year.

A.8 Oil Price Index

For oil prices, we used monthly averages of the Brent Crude series from Datastream.

APPENDIX B: CONSTRUCTION OF REGIONAL DATA SERIES: DOMESTIC AND FOREIGN

Time series observations at the regional level were constructed as weighted averages of corresponding country-specific series as set out in (B.1) and (B.2). Specifically, the regional variables are constructed from country-specific variables using the following (logarithmic) weighted averages:

$$y_{it} = \sum_{\ell=1}^{N_i} w_{i\ell}^0 y_{i\ell t}, \qquad p_{it} = \sum_{\ell=1}^{N_i} w_{i\ell}^0 p_{i\ell t}, \qquad q_{it} = \sum_{\ell=1}^{N_i} w_{i\ell}^0 q_{i\ell t} \tag{B.1}$$

and

$$e_{it} = \sum_{\ell=1}^{N_i} w_{i\ell}^0 e_{i\ell t}, \qquad \rho_{it} = \sum_{\ell=1}^{N_i} w_{i\ell}^0 \rho_{i\ell t}, \qquad m_{it} = \sum_{\ell=1}^{N_i} w_{i\ell}^0 m_{i\ell t}. \tag{B.2}$$

The weights $w_{i\ell}^0$ could be changed at fixed time intervals, say every five years, to capture secular changes in the composition of the regional output. However, changing these weights too frequently could mask the cyclical movements of the regional output being measured. Notice that in constructing the regional variables $y_{it}, p_{it}, e_{it}, \ldots$ from the country-specific variables $y_{i\ell t}, p_{i\ell t}, e_{i\ell t}, \ldots,$ one simply needs to use country-specific variables measured in their domestic currencies. Notice that "$e_{i\ell t}$" stands for the exchange rate of country ℓ in region i, in terms of U.S. dollars.

For weights, we used the GDP shares of each country in the region, computed as that country's PPP-adjusted GDP divided by the total PPP/USD GDP of the region. To avoid using time-varying weights, we chose a relatively recent time period for which PPP data are available, namely 1996.

Not all time series were available for all countries over the entire sample period. As a result, we allow the composition of the regional series to change as data on specific countries become available. For example, if data is not available for a given country over the first few periods in the sample, then a zero weight is attached to this country, with the weights of the remaining countries in the region adjusted to ensure that the sum of the weights add up to unity. Once data become available for the country in question, the weights are redistributed, and the new information is "folded into" the dataset.

Foreign variables are constructed uniquely for each region. For example, foreign money supply, m^*, is different for Western Europe and Latin America. We use the trade shares to appropriately weight the influence of foreign regions on a specified region's economy. Using an interregional trade matrix, we first compute the trade shares for each region with a given country (e.g., the percentage of Argentina's trade originating from Western Europe), and then aggregate across countries based on the trade weights of the countries within the region.

The weights used to aggregate, across countries, the foreign variables need to be constructed with care. Because each starred variable is a weighted average of regional starred variables, if a given region's x variable is not available, then the weighted average must be adjusted to reflect the fact that the foreign variable is not composed of all of the x variables. This can be easily accomplished. For example, suppose that we are computing the German q^* and that $z\%$ of Germany's trade is with Turkey. However, Turkey's equity index is not available. When we take a weighted average of Germany's trading partners' equity indices, we will be effectively weighting only $(1 - z)\%$, because the Turkish index is unavailable. We can then divide our result by $(1 - z)\%$ to yield the appropriate q^* for Germany. Finally, for regions with more than one member country, there exists "intraregional" trade (i.e., trade between countries in the same region) that will not appear in the "foreign" (starred) variables. As such, the weights may sum to less than one.

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