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On the network topology of variance decompositions: Measuring the connectedness of financial firms

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ABSTRACT

We propose several connectedness measures built from pieces of variance decompositions, and we argue that they provide natural and insightful measures of connectedness. We also show that variance decompositions define weighted, directed networks, so that our connectedness measures are intimately related to key measures of connectedness used in the network literature. Building on these insights, we track daily time-varying connectedness of major US financial institutions' stock return volatilities in recent years, with emphasis on the financial crisis of 2007–2008.

"When you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind: it may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the stage of science".
[Kelvin (1891)]

"None of us anticipated the magnitude of the ripple effects".
[Merrill Lynch President Gregory Fleming
on the financial crisis of 2007–2008,
as reported in Lowenstein (2010)]

1. Introduction

Connectedness would appear central to modern risk measurement and management, and indeed it is. It features prominently in key aspects of market risk (return connectedness and portfolio concentration), credit risk (default connectedness), counter-party and gridlock risk (bilateral and multilateral contractual connectedness), and not least, systemic risk (system-wide connectedness). It is also central to understanding underlying fundamental macroeconomic risks, in particular business cycle risk (intra- and inter-country real activity connectedness).

Perhaps surprisingly, then, connectedness remains a rather elusive concept, in many respects incompletely defined and poorly measured. Correlation-based measures remain widespread, yet they measure only pairwise association and are largely wed to linear, Gaussian thinking, making them of limited value in financial-market contexts. Different authors chip away at this situation in different ways. The equi-correlation approach of Engle and Kelly (2012), for example, effectively focuses on average pairwise correlation. The CoVaR approach of Adrian and Brunnermeier (2011) and the marginal expected shortfall (MES) approach of Acharya et al. (2010) and Acharya et al. (2012) go beyond pairwise association, tracking association between individual-firm and overall-market movements, in one direction or the other. The equi-correlation, CoVaR and MES approaches are certainly of interest, but they measure different things, and a unified framework remains elusive.

To address this situation, in this paper we develop and apply a unified framework for conceptualizing and empirically measuring connectedness at a variety of levels, from pairwise through system-wide, using variance decompositions from approximating models.

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We are proud and grateful to be able to build upon the pioneering insights of Halbert L. White Jr., in several ways ranging from the general to the specific. Generally, for example, our connectedness measures are very much linked to and built upon his tradition of dynamic predictive modeling under misspecification.[1] Specifically, in addition, our approach is tightly linked to the graphical (i.e., network) models in which he made pioneering contributions to understanding causal linkages.[2]

We proceed as follows. In Section 2 we introduce the conceptual framework and population connectedness measures. In Section 3 we treat connectedness estimation. In Section 4 we relate our framework and connectedness measures to both the network literature and the systemic risk literature; the relationships turn out to be direct and important. Finally, in Section 5, we apply our framework to study connectedness at all levels among a large set of return volatilities of US financial institutions during the last decade, including during the financial crisis of 2007–2008. We conclude in Section 6.

2. Population connectedness

Our approach to connectedness is based on assessing shares of forecast error variation in various locations (firms, markets, countries, etc.) due to shocks arising *elsewhere*. This is intimately related to the familiar econometric notion of a variance decomposition, in which the forecast error variance of variable i is decomposed into parts attributed to the various variables in the system. We denote by d_{ij}^H the ij-th H-step variance decomposition component; that is, the fraction of variable i's H-step forecast error variance due to shocks in variable j. All of our connectedness measures – from simple pairwise to system-wide – are based on the "non-own", or "cross", variance decompositions, d_{ij}^H, $i, j = 1, \ldots, N$, $i \neq j$. The key is $i \neq j$.

2.1. The population data-generating process

Consider an N-dimensional covariance-stationary data-generating process (DGP) with orthogonal shocks: $x_t = \Theta(L)u_t$, $\Theta(L) = \Theta_0 + \Theta_1 L + \Theta_2 L^2 + \cdots$, $E(u_t u_t') = I$. Note that Θ_0 need not be diagonal. All aspects of connectedness are contained in this very general representation. In particular, contemporaneous aspects of connectedness are summarized in Θ_0, and dynamic aspects in $\{\Theta_1, \Theta_2, \ldots\}$. Nevertheless, attempting to understand connectedness via the potentially many hundreds of coefficients in $\{\Theta_0, \Theta_1, \Theta_2, \ldots\}$ is typically fruitless. One needs a transformation of $\{\Theta_0, \Theta_1, \Theta_2, \ldots\}$ that better reveals and more compactly summarizes connectedness. Variance decompositions achieve this.

2.2. The population connectedness table

The simple Table 1, which we call a *connectedness table*, proves central for understanding the various connectedness measures and their relationships. Its main upper-left $N \times N$ block contains the variance decompositions. For future reference we call that upper-left block a "variance decomposition matrix", and we denote it by $D^H = [d_{ij}^H]$. The connectedness table simply augments D^H with a rightmost column containing row sums, a bottom row containing column sums, and a bottom-right element containing the grand average, in all cases for $i \neq j$.

The off-diagonal entries of D^H are the parts of the N forecast error variance decompositions of relevance from a connectedness perspective; in particular, they measure pairwise directional connectedness. Hence we define the *pairwise directional connectedness* from j to i as

$$C_{i\leftarrow j}^H = d_{ij}^H. \tag{1}$$

Note that in general $C_{i\leftarrow j}^H \neq C_{j\leftarrow i}^H$, so there are $N^2 - N$ separate pairwise directional connectedness measures. They are analogous to bilateral imports and exports for each of a set of N countries. Sometimes we are interested in "net", as opposed to "gross", pairwise directional connectedness. We immediately define *net pairwise directional connectedness* as $C_{ij}^H = C_{j\leftarrow i}^H - C_{i\leftarrow j}^H$.[3] There are $\frac{N^2-N}{2}$ net pairwise directional connectedness measures, analogous to bilateral trade balances.

Now consider not the individual elements of D^H, but rather its off-diagonal row or column sums. Take the first row, for example. The sum of its off-diagonal elements gives the share of the H-step forecast error variance of variable 1 coming from shocks arising in other variables (all other, as opposed to a single other). Hence we call the off-diagonal row and column sums, labeled "from" and "to" in the connectedness table, the total directional connectedness measures. That is, we define *total directional connectedness from others to i* as

$$C_{i\leftarrow\bullet}^H = \sum_{\substack{j=1\\ j\neq i}}^N d_{ij}^H, \tag{2}$$

and *total directional connectedness to others from j* as

$$C_{\bullet\leftarrow j}^H = \sum_{\substack{i=1\\ i\neq j}}^N d_{ij}^H. \tag{3}$$

There are $2N$ total directional connectedness measures, N "to others", or "transmitted", and N "from others", or "received", analogous to total exports and total imports for each of a set of N countries. Just as with pairwise directional connectedness, we are sometimes interested in net total effects. We define *net total directional connectedness* as $C_i^H = C_{\bullet\leftarrow i}^H - C_{i\leftarrow\bullet}^H$. There are N net total directional connectedness measures, analogous to the total trade balances of each of a set of N countries.

Finally, the grand total of the off-diagonal entries in D^H (equivalently, the sum of the "from" column or "to" row) measures

Table 1
Connectedness Table Schematic. See Text for details.

	x_1	x_2	$\cdots$	x_N	From others
x_1	d_{11}^H	d_{12}^H	$\cdots$	d_{1N}^H	$\Sigma_{j=1}^N d_{1j}^H, j \neq 1$
x_2	d_{21}^H	d_{22}^H	$\cdots$	d_{2N}^H	$\Sigma_{j=1}^N d_{2j}^H, j \neq 2$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
x_N	d_{N1}^H	d_{N2}^H	$\cdots$	d_{NN}^H	$\Sigma_{j=1}^N d_{Nj}^H, j \neq N$
To others	$\Sigma_{i=1}^N d_{i1}^H$ $i \neq 1$	$\Sigma_{i=1}^N d_{i2}^H$ $i \neq 2$	$\cdots$	$\Sigma_{i=1}^N d_{iN}^H$ $i \neq N$	$\frac{1}{N}\Sigma_{i,j=1}^N d_{ij}^H$ $i \neq j$

1 See, for example, White (1994).

2 See, for example, White and Chalak (2009).

3 We see gross and net connectedness measures as complements, not substitutes, but we sometimes find net measures of interest and sometimes focus on them in our subsequent empirical analysis. Such net measures are precisely analogous to a trade balance, whether bilateral or multilateral – exports of future uncertainty, less imports of future uncertainty – and they are informative and worthy of study, just as is a trade balance in international economics. We hasten to add, of course, that for some purposes one might be interested in examining individual imports and exports, not just their difference.

total connectedness. We have[4]

$$C^H = \frac{1}{N} \sum_{\substack{i,j=1 \\ i \neq j}}^{N} d_{ij}^H. \tag{4}$$

There is just one total connectedness measure, as total connectedness distills a system into a single number analogous to total world exports or total world imports. (The two are of course identical.)

The connectedness table makes clear how one can begin with the most disaggregated (e.g., microeconomic, firm-level, pairwise-directional) connectedness measures and aggregate them in various ways to obtain macroeconomic economy-wide total directional and total connectedness measures. Different agents may be relatively more interested in one or another of the measures. For example, firm i may be maximally interested in how various others connect to it ($C^H_{i\leftarrow j}$, for various j), or how *all* others connect to it, $C^H_{i\leftarrow\bullet}$. In contrast, regulators might be more concerned with identifying systemically important firms j, in the sense of large total directional connectedness to others from j, $C^H_{\bullet\leftarrow j}$, and they might also be more concerned with monitoring total (system-wide) connectedness C^H.

2.3. Correlated shocks

In the orthogonal reduced-form system discussed thus far, the variance decompositions are easily calculated, because orthogonality ensures that the variance of a weighted sum is simply an appropriately-weighted sum of variances. But reduced-form shocks are rarely orthogonal. To identify uncorrelated structural shocks from correlated reduced-form shocks, one must, inescapably, make assumptions. This is true, for example, with the Cholesky-factor vector autoregression (VAR) identifications popularized by Sims (1980), in any of the scores of subsequent "structural" VAR identifications, in the generalized variance decomposition (GVD) framework of Koop et al. (1996) and Pesaran and Shin (1998), and of course in structural dynamic stochastic general equilibrium environments as surveyed for example by Del Negro and Schorfheide (2011).

Identifying assumptions are just that – *assumptions* – and any set of identifying assumptions may fail. Results based on traditional Cholesky-factor identification, for example, may be sensitive to ordering, as Cholesky-factor identification amounts to assumption of a particular recursive ordering. Many models, moreover, are exactly identified as opposed to over-identified, so that the identifying restrictions cannot be tested. The upshot is that reasonable people may disagree as to their preferred assumption, and they often do. We have nothing new to add; one must make an assumption and move forward conditional upon (and cognizant of) the assumption.

Our own preferences run toward Cholesky and related identifications. They are appealing for our purposes because of their comparatively agnostic data-based spirit. We often find that total connectedness, our most important system-wide summary measure, is robust to Cholesky ordering; that is, the range of total connectedness estimates across orderings is often quite small. Moreover, Swanson and Granger (1997) provide useful methods for testing proposed Cholesky orderings, just as Bernanke (1986) earlier provides tests of structural identifying restrictions.[5]

Nevertheless, there is an intrinsic appeal to order-invariance, which enhances the appeal of GVDs as opposed to Cholesky-based variance decompositions.[6] GVDs were introduced in Pesaran and Shin (1998), which builds on Koop et al. (1996). Like Cholesky-based variance decompositions, GVDs rely on a largely data-based identification scheme, but they are invariant to ordering. In a Cholesky-factor orthogonalization, the first variable in the ordering is affected contemporaneously only by its own innovations, the second variable in the ordering is affected contemporaneously only by innovations of the first and second variables, and so on. GVDs, in contrast, effectively treat each variable as "first in the ordering". They do so not by attempting to orthogonalize shocks, but rather by allowing for correlated shocks while simultaneously accounting for the correlation among them observed historically, under a normality assumption.

Mechanically, the H-step generalized variance decomposition matrix $D^{gH} = [d^{gH}_{ij}]$ has entries

$$d_{ij}^{gH} = \frac{\sigma_{jj}^{-1} \sum_{h=0}^{H-1} (e_i' \Theta_h \Sigma e_j)^2}{\sum_{h=0}^{H-1} (e_i' \Theta_h \Sigma \Theta_h' e_i)}, \tag{5}$$

where e_j is a selection vector with jth element unity and zeros elsewhere, Θ_h is the coefficient matrix multiplying the h-lagged shock vector in the infinite moving-average representation of the non-orthogonalized VAR, Σ is the covariance matrix of the shock vector in the non-orthogonalized VAR, and σ_{jj} is the jth diagonal element of Σ.[7] Because shocks are not necessarily orthogonal in the GVD environment, sums of forecast error variance contributions are not necessarily unity (that is, row sums of D^g are not necessarily unity).[8] Hence we base our generalized connectedness indexes not on D^g, but rather on $\tilde{D}^g = [\tilde{d}^g_{ij}]$, where $\tilde{d}^g_{ij} = \frac{d^g_{ij}}{\sum_{j=1}^N d^g_{ij}}$. By construction $\sum_{j=1}^N \tilde{d}^g_{ij} = 1$ and $\sum_{i,j=1}^N \tilde{d}^g_{ij} = N$. Using $\tilde{D}^g$ we can immediately calculate generalized connectedness measures.

3. Sample connectedness

Clearly C depends on the set of variables x whose connectedness is to be examined, the predictive horizon H for variance decompositions, and the dynamics $A(L)$, so we write $C(x, H, A(L))$.[9] In reality $A(L)$ is unknown and must be approximated (e.g., using a finite-ordered vector autoregression). Recognizing the centrality of the approximating model adopted, we write $C(x, H, A(L), M(L; \theta))$, where $M(L; \theta)$ is a dynamic approximating model with finite-dimensional parameter θ. One hopes that $M(L; \theta)$ is in some sense close to the true population dynamics $A(L)$ for some pseudo-true parameter configuration θ_0, but there is of course no guarantee.

In addition, and crucially, we want to allow for time-varying connectedness, which allows us to move from the static, unconditional, perspective implicitly adopted thus far, to a dynamic, conditional perspective. Time-varying $A(L)$, and hence time-varying

4 Note that we construct total connectedness by taking off-diagonal D^H variation relative to total D^H variation (N), so that C^H is expressed as a decimal share, as with total directional connectedness "from". For the same reason it may also be desirable to scale total directional connectedness "to" by N.

5 We reserve their exploration, however, for future work.

6 Other order-invariant identifications are also possible, such as the symmetric matrix square root, which we hope to explore in future work. We thank Chris Sims for alerting us to that possibility.

7 Note the typo in the original paper of Pesaran and Shin (1998), p. 20. They write σ_{ii}^{-1} but should have written σ_{jj}^{-1}.

8 We now drop the "H" superscripts, because from this point onward they are not needed for clarity.

9 The same holds, of course, for the various directional connectedness measures, so we use $C(x, H, A(L))$ as a stand-in for all our connectedness measures.

connectedness, may arise for a variety of reasons. $A(L)$ may evolve slowly with evolving tastes, technologies and institutions, or it may vary with the business cycle, or it may shift abruptly with financial market environment (e.g., crisis, non-crisis). Whether and how much $A(L)$ varies is ultimately an empirical matter and will surely differ across applications, but in any event it would be foolish simply to assume it is constant. Hence we allow the connection table and all of its elements to vary over time, and we write $C_t(x, H, A_t(L), M(\theta_t))$.

Finally, everything we have written thus far refers to the *population*, whereas in reality we have available only finite *samples* of observed data. That is, we must use *estimated* approximating models, so we write $\widehat{C}_t(x, H, A_t(L), M(\hat{\theta}_t))$, where the data sample runs from $t = 1, \ldots, T$. To economize on notation we henceforth drop $A_t(L)$, because it is determined by nature rather than a choice made by the econometrician, relying on the reader to remember its relevance and simply writing $\widehat{C}_t(x, H, M(\hat{\theta}_t))$. In what follows we successively discuss aspects of x, H and $M(\hat{\theta}_t)$.

3.1. The reference universe, x

Connectedness measurements are defined only with respect to a reference universe, namely the set of x's defining the object of interest to be studied. Choice of x has important implications for the appropriate approximating model; for example, x may (or may not) be strongly serially correlated, conditionally heteroskedastic, or highly disaggregated. Connectedness measurements generally will not, and should not, be robust to choice of reference universe.

Three sub-issues arise, which we call the "x object", the "x choice", and the "x frequency". By x object we refer to the type of x variable studied, typically either returns or return volatilities. By x choice we mean precisely which (and hence how many) x variables are chosen for study.[10] By x frequency we refer to the observational frequency of the x variables (daily, monthly, ...). In this paper the x object is the natural log of realized equity return volatility, the x choice is approximately fifteen major US financial institutions, and the x frequency is daily.[11]

3.2. The predictive horizon, H

Certain considerations in certain contexts may help guide selection of connectedness horizon, H. For example, in risk management contexts, one might focus on H values consistent with risk measurement considerations. $H = 10$, for example, would cohere with the 10-day value at risk (VaR) required under the Basel accord. Similarly, in portfolio management contexts one might link H to the rebalancing period.

The connectedness horizon is important particularly because it is related to issues of dynamic connectedness (in the fashion of contagion) as opposed to purely contemporaneous connectedness. To take a simple pairwise example, shocks to j may impact the forecast error variance of i only with a lag, so that $C_{i\leftarrow j}$ may be small for small H but nevertheless larger for larger H.[12] Intuitively, as the horizon lengthens there may be more chance for connectedness to appear. Thus, in a sense, varying H lets us break connectedness into "long-run", "short-run", etc. More precisely, as H lengthens we obtain a corresponding sequence of conditional prediction error variance decompositions for which the conditioning information is becoming progressively less valuable. In the limit as $H \to \infty$, we obtain an *un*conditional variance decomposition.

In this paper we anchor on a horizon of $H = 12$ days, but we also examine a range of nearby H values. In a sense this provides a "robustness check", but as we argued above, there is no reason why connectedness should be "robust" to H. Instead we view examination of a menu of H values simply as an interesting part of a phenomenological investigation.

3.3. The approximating model, $M(\theta_t)$

A first issue is choice of approximating model class. As discussed previously, many choices are possible, ranging from traditional data-driven VAR approaches, to so-called "structural" VARs, to fully-articulated dynamic stochastic general equilibrium (DSGE) models.

A second issue is how to allow for time-varying connectedness, which is potentially of central interest for risk measurement and management (e.g., over the business cycle, or during financial crises). But connectedness is simply a transformation of model parameters, so allowance for time-varying connectedness effectively means allowance for time-varying parameters in the approximating model. Linear models with time-varying parameters are actually very general nonlinear models, as emphasized in White's Theorem (Granger, 2008).[13]

As with choice of approximating model class, many choices are possible to allow for time-varying parameters. A simple and popular scheme involves use of a rolling estimation window. To track time-varying connectedness in real-time, for example, we might use a uniform one-sided estimation window of width w, sweeping through the sample, at each period using only the most recent w periods to estimate the approximating model and calculate connectedness measures.[14] We write $\hat{C}_t(x, H, M_{t-w:t}(\hat{\theta}))$. The rolling-window approach has the advantages of tremendous simplicity and coherence with a wide variety of possible underlying time-varying parameter mechanisms. Rolling windows do, however, require choice of window width w, in a manner precisely analogous to bandwidth choice in density estimation. In this paper we focus on a VAR(3) approximating model with a one-sided rolling estimation window of $w = 100$ days, but we also explore robustness to alternative choices of w.

4. Relationships to the network and systemic risk literatures

Our connectedness measures turn out to be intimately related both to modern network theory and to modern measures of systemic risk. We now consider both.

4.1. Network connectedness

Networks are everywhere in modern life, from power grids to Facebook. Not surprisingly, research on networks has grown explosively in recent years.[15] A network $\mathcal{N}$ is composed of N nodes

10 In future work, Bayesian estimation may be useful not only to promote parsimony in large-N environments, but also to reduce the probability of spuriously-inflated connectedness measurements for large N.

11 Taking logs converts realized volatilities to approximate normality, as emphasized in Andersen et al. (2003).

12 Such dynamic phenomena, and the rich patterns that are possible, are closely related to aspects of multi-step Granger causality, as treated for example in Dufour and Renault (1998); Dufour and Taamouti (2010), and the references therein.

13 Interestingly, it seems that "White's Theorem" was not published by White. Instead, the theorem is attributed to White in Granger (2008).

14 Alternatively, we might explicitly specify a process for the dynamically evolving model parameters, as is commonly done in a state-space framework using the Kalman filter for estimation.

15 Newman (2010); Jackson (2008) and Easley and Kleinberg (2010) provide fine introductions. Seminal contributions to the characterization, detection and estimation of causal links in networks range from the early work by Clark Glymour and Judea Pearl *inter alia*, as distilled in works such as Glymour et al. (1987, 1993) and Pearl (2000), to more recent work by White and Chalak (2009), *inter alia*.

and L links between nodes. The distance s_{ij} between two nodes i and j is the smallest number of links that must be traversed to go from i to j. $\mathcal{N}$ is connected if $s_{ij} \leq N-1, \forall i,j$, and one is naturally led to think about measures of the strength of network connectedness. That is, presumably two connected networks need not be equally strongly connected. But then deep questions arise. Just what *is* strength of network connectedness? Is it a pairwise or system-wide concept, or both, or neither? How, if at all, might it be related to the notion of connectedness that we have proposed independently and thus far emphasized, based on variance decompositions?

To approach the issue of measuring network connectedness, we need to analyze the mathematical structure of networks a bit more deeply. A network is simply an $N \times N$ adjacency matrix A of zeros and ones, $A = [A_{ij}]$, where $A_{ij} = 1$ if nodes i and j are linked, and $A_{ij} = 0$ otherwise. Note that A is symmetric, because if i and j are connected, then so too must be j and i. Mathematically (i.e., algebraically), the adjacency matrix A *is* the network, and all network properties are embedded in A. Hence any sensible connectedness measure must be based on A. Nevertheless, there is no single, all-encompassing measure, and several have been proposed.[16] The most important and popular by far – as well as the most useful for our purposes – are based on the idea of node degree (and a closely-related concept, network diameter), to which we now turn.

4.1.1. Degree and diameter

A node's degree is its number of links to other nodes. Immediately the degree of node i is

$$\delta_i = \sum_{j=1}^{N} A_{ij} = \sum_{j=1}^{N} A_{ji}. \tag{6}$$

We can of course examine the pattern of degrees across nodes. The degree distribution is the probability distribution of degrees across nodes. It is a discrete univariate distribution with support $0, \ldots, (N-1)$, and aspects of its shape (location, scale, skewness, tail thickness, etc.) are closely linked to aspects of network behavior.[17] As regards the aspect of network behavior that concerns us – connectedness – the location of the degree distribution is obviously key, and the standard location measure is of course the mean. Hence the mean of the degree distribution (mean degree) has emerged as a canonical benchmark measure of overall network connectedness. The larger the mean degree, the greater is the overall network connectedness.

The just-described adjacency matrix and degree distribution might more precisely be called "1-step", as the links are direct. However, even if i is not directly linked to j, i may be linked to k, and k to j, so that i and j are linked at a distance of two steps rather than one. The distinction between 1-step and multi-step adjacency emphasizes distance. Recall that, as introduced earlier, the distance s_{ij} between two nodes i and j is the smallest number of links that must be traversed to go from i to j. Distance is a two-node property, in contrast to degree, which is a single-node property. Closely related to the idea of distance is the idea of diameter. The diameter of a network is the maximum distance between any two nodes, $s_{max} = max_{i,j} s_{ij}$. Diameter is another canonical benchmark measure of overall network connectedness. The smaller the network diameter, the greater is the overall connectedness.

A beautiful large-N approximation relates network diameter, network mean degree and network size in Erdős–Rényi random networks (Erdős and Rényi, 1959)[18]:

$$s_{max} \approx \frac{\ln N}{\ln E(\delta)}. \tag{7}$$

This "network diameter grows only as $\ln N$" approximation is typically introduced as a mathematically-precise characterization of the "small-world" phenomenon, namely that diameters tend to be small even for huge networks.[19] For our purposes, however, it is useful because it emphasizes in a very precise way the importance of the mean degree as a measure of network connectedness. As we shall now see, our earlier-proposed connectedness measures are intimately related to certain network node degrees and mean degree.

4.1.2. Variance decompositions as weighted, directed networks

Interestingly, it turns out that our connectedness measures, early variants of which were proposed in Diebold and Yılmaz (2009) independently of the network literature, are closely related to aspects of network connectedness. Indeed we are now in a position to notice that *variance decompositions are networks*. More precisely, the variance decomposition matrix D, which defines our connectedness table and all associated connectedness measures, is a network adjacency matrix A. Hence network connectedness measures may be used in conjunction with variance decompositions to understand connectedness among components.

The networks defined by variance decompositions, however, are rather more sophisticated than the classical network structures sketched thus far. First, the adjacency matrix A (variance decomposition matrix D) is not filled simply with 0–1 entries; rather, the entries are weights, with some potentially strong and others potentially weak. Second, the links are directed; that is, the strength of the ij link is not necessarily the same as that of the ji link, so the adjacency matrix is generally not symmetric. Third, there are constraints on the row sums of A. In particular, each row must sum to 1 because the entries are variance shares. Hence we write the diagonal elements as $A_{ii} = 1 - \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}$. Note in particular that the diagonal elements of A are no longer 0.

Weighted, directed versions of the earlier-introduced network connectedness statistics are readily defined, including degrees, degree distributions, distances and diameters. For example, node degrees are now obtained not by summing zeros and ones, but rather by summing weights in [0, 1]. Moreover, there are now "to-degrees" and "from-degrees", corresponding to row sums and column sums.[20] The from-degree of node i is $\delta_i^{from} = \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}$. The from-degree distribution is the probability distribution of from

[16] Bech and Atalay (2011) and Adamic et al. (2010) provide good reviews and financial applications of network-theoretic connectedness measures.

[17] The support of $0, \ldots, (N-1)$ stems from our adoption of the standard convention of writing $A_{ii} = 0, \forall i$.

[18] See, for example, Newman (2010), p. 420. Erdős–Rényi random networks have the simplest imaginable probabilistic model of link formation: independent Bernoulli trials with fixed probability θ. Hence the degree distribution is binomial, $f(\delta) = \binom{N-1}{\delta}\theta^{\delta}(1-\theta)^{N-1-\delta}$. Erdős–Rényi random networks have emerged as a canonical benchmark, but they are sometimes poor descriptions of real-world networks, due for example to strategic aspects of link formation such as clustering, which refers to the fact that in real networks two "people" with a common "friend" are more likely to be friends than two randomly-selected people. Interestingly, however, Watts and Strogatz (1998) have shown that the "network diameter grows only as $\ln N$" approximation nevertheless holds in networks with small clusters of linked nodes with just a few long-range links.

[19] For example, for $N = 300{,}000{,}000$ (roughly the US population) and mean degree $E(\delta) = 20$, network diameter is still small ($s_{max} \approx 6$). That is, even if every person in the US is linked to only twenty others on average, then the approximate maximum number of steps needed to link any two people is nevertheless only six.

[20] Our to-degrees and from-degrees are often called "out-degrees" and "in-degrees" in the network literature.

degrees across nodes. It is a univariate distribution with support on [0, 1]. Similarly, the to-degree of node j is $\delta_i^{to} = \sum_{\substack{i=1\\i\neq j}}^{N} A_{ij}$. The to-degree distribution is the probability distribution of to degrees across nodes. It is a univariate distribution with support on [0, N].

By now the relationships between our earlier-defined connectedness measures and those used in the network literature should be apparent. First, our total directional connectedness measures $C_{i\leftarrow\bullet}$ and $C_{\bullet\leftarrow j}$ are precisely the from-degrees and to-degrees, respectively, associated with the nodes of the weighted directed network D. Second, our total connectedness measure C is simply the mean degree of the network D (to or from—it is the same either way, because the sum of all row sums must equal the sum of all column sums).

4.2. Systemic risk measurement

There is no single definition of systemic risk, but the defining characteristic is that systemic risk involves aspects of market-wide connectedness, one way or another. Here we introduce two of the most important systemic risk measures, and we show how our network-based connectedness measures are related.

4.2.1. Marginal expected shortfall and expected capital shortfall

Marginal expected shortfall (MES) for firm j is

$$MES^{j|mkt}_{T+1|T} = E_T\left[r_{j,T+1}|\mathbb{C}\left(r_{mkt,T+1}\right)\right], \tag{8}$$

where $r_{mkt,T+1}$ denotes the overall market return, and $\mathbb{C}\left(r_{mkt,T+1}\right)$ denotes a market-wide extreme event, such as the market return falling below some threshold. $MES^{j|mkt}$ tracks the sensitivity of firm j's return to a market-wide extreme event, thereby providing a simple market-based measure of firm j's fragility.

Ultimately, however, we are interested in assessing the likelihood of firm distress, and the fact that a firm's expected return is sensitive to market-wide extreme events – that is, the fact that its MES is large – does not necessarily mean that market-wide extreme events are likely to place it in financial distress. Instead, the distress likelihood should depend not only on MES, but also on how much capital the firm has on hand to buffer the effects of adverse market moves. This consideration raises the idea of expected capital shortfall (*ECS*), which is closely related to, but distinct from, MES. *ECS* is the expected *additional capital* needed in case of a systemic market event. Clearly *ECS* should be related to MES, and Acharya et al. (2010) show that in a simple model the two are linearly related,

$$ECS^{j|mkt}_{T+1|T} = a_{0j} + a_{1j}MES^{j|mkt}_{T+1|T}, \tag{9}$$

where a_{0j} depends on firm j's "prudential ratio" of asset value to equity as well as its debt composition, and a_{1j} depends on firm j's prudential ratio and initial capital. Building on the theory of Acharya et al. (2010); Brownlees and Engle (2011) propose and empirically implement $ECS^{j|mkt}_{T+1|T}$ as a measure of firm j's systemic risk exposure to the market at time T, with overall systemic risk then given by $\sum_{j=1}^{N} ECS^{j|mkt}_{T+1|T}$.[21]

4.2.2. CoVaR and ΔCoVaR

In the previous section we introduced MES and *ECS*, which measure firm systemic risk *exposure* by conditioning firm events on market events. Here we introduce CoVaR, which works in the opposite direction, measuring firm systemic risk *contribution* by conditioning market events on firm events.

[21] Note that *ECS* has the virtue of being readily aggregated across firms.

First recall the well-known concept of value at risk (VaR). Firm j's 1-step-ahead conditional VaR at level p is the value of $\mathrm{VaR}^{p,j}_{T+1|T}$ that solves

$$\Pr_T\left(r_{j,T+1} < -\mathrm{VaR}^{p,j}_{T+1|T}\right) = p. \tag{10}$$

It is a natural next step, following Adrian and Brunnermeier (2011), then to define firm j's 1-step-ahead "CoVaR" at level p conditional on a particular outcome for firm i, say $\mathbb{C}\left(r_{i,T+1}\right)$, as the value of $\mathrm{CoVaR}^{j|i}_{T+1|T}$ that solves

$$\Pr_T\left(r_{j,T+1} < -\mathrm{CoVaR}^{j|i}_{T+1|T} \mid \mathbb{C}\left(r_{i,T+1}\right)\right) = p. \tag{11}$$

Because $\mathbb{C}\left(r_{i,T+1}\right)$ is not in the time-T information set, CoVaR will be different from the regular time-T conditional VaR. The leading choice of conditioning outcome, $\mathbb{C}\left(r_{i,T+1}\right)$, is that firm i exceeds its VaR, or more precisely that $r_{i,T+1} < -\mathrm{VaR}^{p,i}_{T+1|T}$. As such, CoVaR is well-suited to measure tail-event linkages between financial institutions.

A closely-related measure, $\Delta\mathrm{CoVaR}^{j|i}_{T+1|T}$ (read "Delta CoVaR"), is also of interest. It measures the difference between firm-j VaR when firm i is "heavily" stressed and firm-j VaR when firm i experiences "normal" times. More precisely,

$$\Delta\mathrm{CoVaR}^{j|i}_{T+1|T} = \mathrm{CoVaR}^{j|\mathrm{VaR}(i)}_{T+1|T} - \mathrm{CoVaR}^{j|Med(i)}_{T+1|T}, \tag{12}$$

where $\mathrm{CoVaR}^{j|\mathrm{VaR}(i)}_{T+1|T}$ denotes firm-j VaR when firm i's return breaches its VaR, and $\mathrm{CoVaR}^{j|Med(i)}_{T+1|T}$ denotes firm-j VaR when firm i's return equals its median.

A direct extension lets us progress to the more interesting case of firm i's overall systemic risk contribution, as opposed to just firm i's contribution to firm j. We simply set $j = mkt$, so that $\Delta\mathrm{CoVaR}^{mkt|i}_{T+1|T}$ then measures the difference between market VaR conditional on firm i experiencing an extreme return, and market VaR conditional on firm i experiencing a normal return. Hence $\Delta\mathrm{CoVaR}^{mkt|i}_{T+1|T}$ measures the contribution of firm i to overall systemic risk, $\sum_{i=1}^{N} \Delta\mathrm{CoVaR}^{mkt|i}_{T+1|T}$.

4.2.3. Network connectedness, MES and CoVaR

The MES and CoVaR approaches address certain aspects of connectedness, as they track association between individual-firm and overall-market movements. Moreover, both MES and CoVaR are weighted and directional, just as with our connectedness measurement framework. For example, $\mathrm{CoVaR}^{j|i}_{T+1|T}$ tracks effects from i to j, whereas $\mathrm{CoVaR}^{i|j}_{T+1|T}$ tracks effects from j to i, and in general $\mathrm{CoVaR}^{j|i}_{T+1|T} \neq \mathrm{CoVaR}^{i|j}_{T+1|T}$. Hence one suspects that MES and CoVaR should be related to our various connectedness measures for weighted directed networks, yet simultaneously, MES and CoVaR measure different things.

The tension is resolved by noting that our from- and to-degrees measure aspects of systemic risk similar to those tracked by MES and CoVaR, respectively. From-degrees measure exposures of individual firms to systemic shocks *from* the network, in a fashion precisely analogous to $MES^{j|mkt}_{T+1|T}$. To-degrees measure contributions of individual firms *to* systemic network events, in a fashion precisely analogous to $\Delta\mathrm{CoVaR}^{mkt|i}_{T+1|T}$. Moreover, our total degree aggregates firm-specific systemic risk across firms, providing a natural measure of total system-wide systemic risk not unlike total *ECS*.

Our framework, then, unifies MES and CoVaR insofar as it makes clear that they are closely related to different directional aggregations of a certain weighted directed network. But it also goes farther, insofar as it starts with greater granularity, furnishing information at the pairwise level, and it finishes with greater aggregation, furnishing information at the network-wide level, which moreover is consistent regardless of whether one aggregates "from" or "to".

5. Connectedness of US financial institutions

Thus far we have introduced tools for connectedness measurement and emphasized their relationship to the structure of weighted directed networks. We now put those tools to work, using them to monitor and characterize the evolution of connectedness among major US financial institutions before and during the 2007–2008 financial crisis. Understanding such financial connectedness is of interest not only in terms of understanding financial crises, but also in terms of understanding the business cycle, as the financial system's health has important implications for real economic health.

We remain agnostic as to how connectedness arises; rather, we take it as given and seek to measure it correctly for a wide range of possible underlying causal structures. Obviously there are tradeoffs, but we prefer an approach that potentially achieves much under minimal assumptions, in contrast to a more deeply structural approach that in principle could achieve even more, but only under heroic assumptions.

We proceed in four steps. First, in Section 5.1, we describe the data that we use to measure financial institution connectedness. Next, in Section 5.2, we perform a full-sample (static) analysis, in which we effectively characterize average, or unconditional, connectedness. This is of intrinsic interest, and it also sets the stage for Section 5.3, where we perform a rolling-sample (dynamic) analysis of conditional connectedness. Our ultimate interest lies there; we monitor high-frequency (daily) connectedness as conditions evolve, sometimes gradually and sometimes abruptly. Finally, in Section 5.4, we "zoom in" on financial institution connectedness during the global financial crisis of 2007–2008.

5.1. Firm-level stock return volatility data

Financial institutions are connected directly through counterparty linkages associated with positions in various assets, through contractual obligations associated with services provided to clients and other institutions, and in many other ways. High-frequency analysis of financial institution connectedness therefore might seem to require high-frequency balance sheet and other information, which is generally unavailable. Fortunately, however, stock market returns and return volatilities are available, which reflect forward-looking assessments of many thousands of smart, strategic and often privately-informed agents as regards precisely the relevant sorts of connections. We use that data to measure connectedness and its evolution.[22]

We study *volatility* connectedness, for at least two reasons. First, if volatility tracks investor fear (e.g., the volatility index, "VIX", traded on the Chicago Board Options Exchange, is often touted as an "investor fear gauge"), then volatility connectedness is the "fear connectedness" expressed by market participants as they trade.[23] We are interested in the level, variation, paths, patterns and clustering in precisely that fear connectedness. Second, volatility connectedness is of special interest because we are particularly interested in crises, and volatility is particularly crisis-sensitive.

Volatility is latent and hence must be estimated. We use realized volatility, which has received significant attention in recent years.[24] For a given firm on a given day, we construct daily realized return volatility using high-frequency intra-day data from the Trade and Quote (TAQ) database. In particular, we calculate daily realized volatility as the sum of squared log price changes over the 78 5-min intervals during trading hours, from 09:00–12:00 and 13:00–16:30.

We treat realized volatility as the object of direct interest, as in Andersen et al. (2003).[25] This is appropriate because for the large, heavily-traded firms that we examine, five-minute sampling is frequent enough largely to eliminate measurement error, yet infrequent enough such that microstructure noise (e.g., due to bid–ask bounce) is not a concern. In addition, and importantly, realized volatility actually *is* an object of direct interest, traded in the volatility swap markets, in contrast to underlying quadratic variation or any other object that realized volatility may or may not be construed as estimating.

Volatilities tend to be strongly serially correlated—much more so than returns, particularly when observed at relatively high frequency. We capture that serial correlation using vector-autoregressive approximating models, as described earlier. Volatilities also tend to be distributed asymmetrically, with a right skew, and approximate normality is often obtained by taking natural logarithms. Hence we work with log volatilities. This is helpful not only generally, as normality-inducing transformations take us into familiar territory, but also specifically as we use generalized variance decompositions (Koop et al., 1996; Pesaran and Shin, 1998), which invoke normality.

5.2. Static (full-sample, unconditional) analysis

Here we study stock return volatilities for thirteen major US financial institutions that survived the crisis of 2007–2008. In Table 2 we list the firms, tickers, market capitalization before and after the crisis, and critical episodes/dates during the crisis. Our

Table 2
US Financial Institution Detail. Com Bank denotes a commercial bank, and Inv Bank denotes an investment bank. Market capitalizations are in billions of US dollars. Fannie Mae and Freddie Mac were placed into government conservatorship on September 7, 2008, and AIG began government ownership on September 17, 2008.

Institution	Ticker	Business	Market Cap.	
			12/29/06	12/31/09
J.P. Morgan	JPM	Com Bank	169	171
Wells Fargo	WFC	Com Bank	121	137
Bank of America	BAC	Com Bank	241	131
Citigroup	C	Com Bank	274	76
US Bancorp	USB	Com Bank	64	43
Bank of New York Mellon	BK	Com Bank	30	34
PNC Group	PNC	Com Bank	22	24
American Express	AXP	Credit Cards	74	49
Goldman Sachs	GS	Inv Bank	86	86
Morgan Stanley	MS	Inv Bank	85	40
Fannie Mae	FNM	Mortgages	59	1.3
Freddie Mac	FRE	Mortgages	47	0.9
AIG	AIG	Insurance	187	4

22 Some take issue with market data, and certainly we would not argue that all markets are perfect at all times — far from it. But we do feel strongly that it is harder to fool markets than to fool regulators. Other leading frameworks also take a market-based approach, as for example with marginal expected shortfall and CoVaR. Hautsch et al. (2012) also make good use of market-based information.

23 The VIX, short for "volatility index", is traded on the Chicago Board Options Exchange. It tracks the market volatility implied by traded S&P 500 options.

24 For surveys see Andersen et al. (2010, 2013).

25 This contrasts with an alternative approach that views realized volatility not as the direct object of interest, but rather as an estimate of underlying quadratic variation. In that case one might want to acknowledge estimation error explicitly, as in Hansen and Lunde (2014).

Table 3
Full-Sample Connectedness Table. The sample is May 4, 1999 through April 30, 2010, and the predictive horizon is 12 days. The *ij*-th entry of the upper-left 13 × 13 firm submatrix gives the *ij*-th pairwise directional connectedness; i.e., the percent of 12-day-ahead forecast error variance of firm *i* due to shocks from firm *j*. The rightmost (FROM) column gives total directional connectedness (from); i.e., row sums (from all others to *i*). The bottom (TO) row gives total directional connectedness (to); i.e., column sums (to all others from *j*). The bottommost (NET) row gives the difference in total directional connectedness (to–from). The bottom-right element (in boldface) is total connectedness (mean "from" connectedness, or equivalently, mean "to" connectedness).

	AXP	BAC	BK	C	GS	JPM	MS	PNC	USB	WFC	AIG	FNM	FRE	**FROM**
AXP	20.0	8.5	7.1	10.3	5.8	9.8	8.8	5.1	8.0	7.8	3.2	2.6	3.0	80.0
BAC	8.3	19.1	6.0	10.6	5.8	8.0	7.4	6.1	7.1	9.2	4.2	3.5	4.6	80.9
BK	8.4	8.3	18.8	8.4	6.2	9.3	8.5	5.7	8.4	8.3	4.2	2.4	3.0	81.2
C	9.5	9.6	5.4	20.4	4.9	8.7	7.8	5.2	7.0	8.0	5.4	3.5	4.7	79.6
GS	8.2	8.6	6.8	7.6	22.1	8.8	13.3	4.0	6.0	7.6	2.4	1.9	2.6	77.9
JPM	10.2	8.6	7.1	10.6	6.2	18.8	9.5	5.2	7.8	7.3	3.6	2.5	2.6	81.2
MS	9.2	8.3	7.1	8.9	9.8	9.7	20.5	4.2	5.5	7.1	3.4	2.8	3.6	79.5
PNC	7.7	8.8	7.4	8.5	4.6	7.6	6.6	18.1	7.6	8.8	5.2	4.2	4.9	81.9
USB	9.3	9.9	7.6	9.9	5.7	8.7	6.4	5.4	20.1	8.5	4.3	1.6	2.7	79.9
WFC	8.3	10.2	6.5	9.8	6.2	7.6	7.1	5.9	7.3	18.0	3.8	3.8	5.3	82.0
AIG	5.3	7.3	4.9	8.8	2.6	5.2	4.9	6.2	6.0	5.6	27.5	6.6	9.0	72.5
FNM	4.2	5.4	2.5	6.0	2.3	3.5	3.8	5.5	1.9	6.8	6.5	29.6	22.0	70.4
FRE	4.3	6.3	2.9	6.5	2.6	3.3	4.1	5.2	2.9	7.3	7.4	17.6	29.6	70.4
TO	92.9	99.7	71.3	106.1	62.7	90.2	88.2	63.7	75.5	92.2	53.8	53.1	68.1	**78.3**
NET	13.0	18.8	−9.9	26.5	−15.2	8.9	8.7	−18.2	−4.4	10.2	−18.7	−17.4	−2.3	

sample includes seven commercial banks, two investment banks, one credit card company, two mortgage finance companies and one insurance company. Stocks of all firms except Fannie Mae and Freddie Mac were included in the S&P500 prior to the sub-prime crisis of 2007.

Our sample begins in May 1999 and ends in April 2010. Starting in 1999 allows us to include among our firms Goldman Sachs, Morgan Stanley and US Bancorp, all of which went public in the late 1990s. Our sample also spans several important financial market episodes in addition to the crisis of 2007–2008.[26] These include the dot-com bubble collapse of 2000, the Enron scandal of October 2001, and the WorldCom/MCI scandal and bankruptcy of July 2002. Hence we can not only assess connectedness of our firms during the crisis of 2007–2008, but also compare and contrast connectedness during other episodes. We include AIG because it was a major supplier of "financial insurance" in the 2000s, selling credit default swaps (CDSs) through its AIG Financial Products arm in London, which featured prominently in the financial crisis of 2007–2008.

The full-sample connectedness table appears as Table 3. Many features are notable. Some blocks of high pairwise directional connectedness appear, especially for the government-sponsored enterprises (Freddie and Fannie) and various investment banks. The diagonal elements (own connectednesses) tend to be the largest individual elements of the table, but total directional connectedness (from others or to others) tends to be much larger, and total connectedness is a very high 78%. In addition, the spread of the "from" degree distribution is noticeably less than that of the "to" degree distribution.

Let us discuss some of the features of the connectedness table at greater length, beginning with the pairwise directional connectedness measures, $\hat{C}^H_{i\leftarrow j}$, which are the off-diagonal elements of the upper-left 13 × 13 submatrix. The highest observed pairwise connectedness is from Freddie Mac to Fannie Mae ($\hat{C}^H_{FNM\leftarrow FRE} = 22\%$). In return, the pairwise connectedness from Fannie Mae to Freddie Mac ($\hat{C}^H_{FRE\leftarrow FNM} = 17.6\%$) is second-highest. The two mortgage finance companies were viewed as twins by the markets, so it is reasonable that their pairwise connectedness measures are quite high.

The next largest pairwise directional connectedness is from Morgan Stanley to Goldman Sachs ($\hat{C}^H_{GS\leftarrow MS} = 13.3\%$), the two top investment banks that survived the 2007–08 financial crisis.[27] Although the connectedness from Goldman Sachs to Morgan Stanley is also high ($\hat{C}^H_{MS\leftarrow GS} = 9.8\%$), in net terms the directional connectedness takes place from Morgan Stanley to Goldman Sachs stock ($\hat{C}^H_{GS,MS} = 3.5\%$).

The highest values of pairwise directional connectedness among the commercial bank stocks are from Citigroup, on the one hand, and Bank of America and J.P. Morgan, on the other ($\hat{C}^H_{BAC\leftarrow C} = \hat{C}^H_{JPM\leftarrow C} = 10.6\%$). The high value of pairwise connectedness from Citigroup to either Bank of America and/or J.P. Morgan shows that, being the worst-hit institution among the top five commercial banks, Citigroup spreads its troubles to other top commercial banks.

As we have seen above, Fannie Mae and Freddie Mac are tightly connected to each other, and tightly connected with AIG as well. Those three institutions had many difficulties during the 2007–08 financial crisis and could have gone bankrupt had the US government not intervened in financial markets in September 2008. Pairwise directional connectedness of the stocks of those institutions with the stocks of other financial institutions tends to be much lower than connectedness of other bank stocks in our sample.

The row sum of the pairwise connectedness measures results in the total directional connectedness from others to each of the thirteen stocks (see Section 2). In other words, the "FROM" column measures the share of volatility shocks received from other financial firm stocks in the total variance of the forecast error for each stock. By definition, it is equal to 100% minus the own share of the total forecast error variance. As the own-effects (diagonal elements of the matrix) range between 18% and 30%, the total directional connectedness in the "FROM" column ranges between 70% and 82%.

Similarly, the column sum of all pairwise connectedness measures results in the corresponding stock's total directional connectedness to others. As each stock's contribution to others' forecast error variances is not constrained to add up to 100%, entries in the "TO" row can exceed 100%. While the financial stocks are largely similar in terms of receiving volatility shocks from others, they are highly differentiated as transmitters of volatility shocks to others. The stark difference between the distributions of the two connectedness measures is clearly observed in their respective empirical

26 The 2007–2008 crisis may itself be split into the sub-prime/liquidity crisis of 2007 and the financial crisis of 2008.

27 Because the other three investment banks ceased to exist in 2008, they are not included in the full-sample connectedness table.

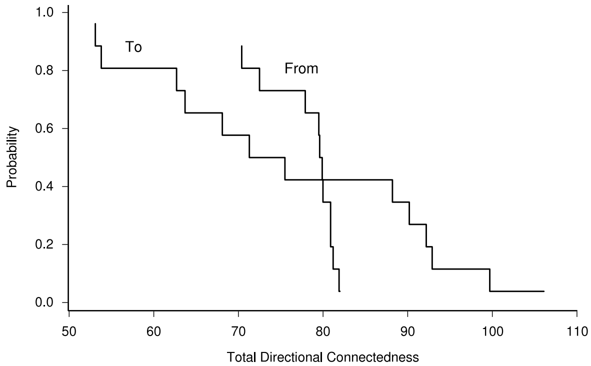


Fig. 1. Full-Sample Total Directional Connectedness: Empirical Survivor Functions. We plot the empirical survivor functions for total directional connectedness "to" others and "from" others. The predictive horizon for the underlying variance decomposition is 12 days.

survivor functions presented in Fig. 1. Compared to the very steep survivor function defined over a narrow range for the connectedness from others, the survivor function for the connectedness to others is quite flat and defined over a wider range. Starting at a minimum of 70% for Fannie and Freddie and increasing only up to a maximum of 82% for Wells Fargo and PNC Group, the total directional connectedness from others is distributed rather tightly. The total directional connectedness to others, on the other hand, varies from a low of 53% for Fannie Mae, to all the way up to 106% for Citigroup: a range of 53 points for the connectedness to others compared to a range of just 12 points for the connectedness from others.

The largest commercial banks (as of 2010) were the ones that have the highest values of connectedness (all exceeding 90%) to others. Being the most vulnerable among them, Citigroup generated a total directional connectedness measure of 106% to others. Besides the top four commercial banks, American Express also generated significant (93%) volatility connectedness to others.

The difference between the total directional connectedness to others and the total directional connectedness from others gives the net total directional connectedness to others ($\hat{C}_i^H = \hat{C}^H_{\bullet\leftarrow i} - \hat{C}^H_{i\leftarrow\bullet}$). In terms of the net total directional connectedness Citigroup (26.5%) leads the way, followed by Bank of America (18.8%), American Express (13%), and J.P. Morgan (8.9%). AIG (−19%), PNC Group (−18%), Fannie Mae (−17%), Goldman Sachs (−15%) and Bank of New York Mellon (−10%) are the financial institutions with negative values of net total directional connectedness to others.

Finally, with a value of 78.3% the measure of total connectedness among the thirteen financial stocks is higher than the total connectedness measures we obtained in other settings, such as the connectedness among different asset classes, or among international stock markets. Given the large number of stocks included in the sample, there is a high degree of connectedness for the full sample. As we will see below there is always a high degree of connectedness even during tranquil times. There is another reason for the total connectedness for a set of financial stocks to be higher than for a set of major national stock markets around the world or for a set of asset classes in the US. As the institutions included in our analysis are all operating in the finance industry, both industry-wide and macroeconomic shocks affect each one of these stocks one way or the other. As some of these institutions and their stocks are more vulnerable to external and/or industry-wide shocks than others, they are likely to be transmitting these shocks to other financial stocks, generating a higher degree of connectedness to others. Obviously, to the extent that they have important implications for the rest of the industry, idiosyncratic volatility shocks are also transmitted to other stocks. For that reason, compared to a similar number of stocks from different industries, the connectedness for a group of stocks in the finance industry is likely to be higher. It is also likely to be higher compared to the connectedness for a group of global markets, as these markets are not subject to common shocks as frequently as the stocks from the finance industry.[28]

5.3. Dynamic (rolling-sample, conditional) analysis

The just-completed analysis of full-sample connectedness provides a good characterization of "average" or "unconditional" aspects of each of the connectedness measures, yet by construction it is silent as to connectedness *dynamics*. In this sub-section we provide a dynamic analysis by using rolling estimation windows. We include the same thirteen financial institutions that we included in our earlier full-sample analysis.[29]

In contrast to our theoretical discussion in Section 2, as well as our static empirical analysis in Section 5.2, in which we progressed from "micro to macro" – that is, from pairwise connectedness, to total directional connectedness, to total connectedness – here it proves useful to proceed in reverse order, from macro to micro. We start our dynamic analysis with total connectedness, and then we move to various levels of disaggregation (total directional and pairwise directional). Finally, we also provide a brief assessment of the robustness of our results to choices of tuning parameters and alternative identification methods.

5.3.1. Total

In Fig. 2 we plot total volatility connectedness over 100-day rolling-sample windows. From a bird's-eye perspective, the total connectedness plot in Fig. 2 has some revealing patterns. It has two big cycles; one starting in late-2000 and ending in mid-2003, whereas the second coincides with the development of the global financial crisis from early 2007 all the way to the end of 2009. The first cycle coincides with the burst of the dot-com bubble, followed by the downward spiral in the Nasdaq and other stock exchanges and the 2001 recession. Even if the recession was over in early 2002, the MCI WorldCom scandal of mid-2002 kept the volatility of the financial stocks and their connectedness high for another year. The second cycle started at the end of February 2007. With the first signs of the sub-prime crisis, the total volatility connectedness index jumped up from a low of around 56% in February 2007 to reach close to 90% in August 2007 and stayed above 80% until mid-2009.

In between the two big cycles of the total connectedness lie three smaller, but not necessarily negligible, cycles. We will discuss each of these cycles along with the events that possibly led to them. Before doing so, let us point to another fact that emerges from the total connectedness plot. From 1999 to 2007, whenever the total connectedness increased to a higher level, it always came back down to the 55%–65% range as the sample windows are rolled to leave that episode behind. Following the 2007–08 financial crisis, the total connectedness index stayed well above this range as of the end of April 2010, even though the financial crisis had ended almost a year before.

Earlier on in our sample, developments in the tech-heavy Nasdaq stock exchange influenced the behavior of the total volatility connectedness among the financial stocks. Starting in March 2000, the so-called dot-com bubble finally started to burst. The bursting of the dot-com bubble had a serious impact on the total volatility connectedness of financial stocks. In March

28 We have in mind a comparison with the total connectedness indexes reported in Diebold and Yılmaz (2009, 2012).

29 In the next sub-section we specifically focus on the 2007–08 financial crisis and include the remaining four institutions (Bear Stearns, Lehman Brothers, Merrill Lynch and Wachovia), all of which ceased trading during the crisis.

Fig. 2. Rolling Total Connectedness. The rolling estimation window width is 100 days, and the predictive horizon for the underlying variance decomposition is 12 days.

2000, the volatility connectedness index increased by 7 percentage points. Despite short spells of recovery, troubles of the internet stocks continued for some time and solid signs of an imminent recession appeared on the horizon. The volatility in the bank stocks increased rapidly over this period, and so did the total volatility connectedness. From a low of 60% in early September, the connectedness index increased to 75% by mid-January 2001 and further to surpass 80% by early May 2001.

The Federal Reserve's intervention, by way of lowering the fed funds target rate by 2.5 percentage points in the first five months of 2001, helped stem the decline in the Nasdaq and other markets toward the second and third quarters of 2001. Total connectedness declined to 71% by early September 2001. However, 9/11 terrorist attacks worsened market sentiment again. Even though the markets were closed for a week after the terrorist attacks, the total connectedness among the financial stocks jumped 10 percentage points in the week it was reopened. The total connectedness stayed around 80% as long as the data for 9/11 were included in the rolling-sample windows.

After the Enron scandal of late 2001, which did not have much impact on financial stocks, another corporate scandal rocked the US financial markets toward the end of June 2002. This time around it was the bankruptcy of the MCI WorldCom, which was once the second-largest long distance phone company in the US. Unlike the Enron scandal, the MCI WorldCom scandal had a serious impact on major bank stocks. All major US banks had credit positions with MCI WorldCom and hence they all suffered losses when the company declared bankruptcy.

Following the bankruptcy, the total connectedness among the major financial institutions jumped from 72% to reach 85% in July 2002, the highest level achieved from the beginning of the sample. However, being an isolated source of loss for the banks, the scandal's impact on the financial system as a whole could be contained. As of the end of 2002 total connectedness subsided very quickly to pre-July 2002 levels. After a brief increase following the invasion of Iraq in March 2003, the total connectedness declined to 58% in August 2003.

From August 2003 to February 2007, the total connectedness index went through three smaller cycles, during which it moved within the 55%–80% range. The first cycle lasted from August 2003 to March 2005; the second from April 2005 to February 2006; and the third from March 2006 to February 2007. The three cycles mostly coincide with the tightening of monetary policy and its impact on the behavior of long-term interest rates.[30]

5.3.2. *Total directional*

The dynamic analysis of total connectedness gave us a clear understanding of the factors influencing the volatility connectedness across major US financial stocks over the 1999–2010 period. Keeping this analysis in the back of our minds, we can now focus on the dynamics of directional connectedness over time.

Fig. 3 presents the time series of total directional connectedness ("to" and "from" degrees) separately for each firm. The plots for total directional connectedness "to" others are presented in the upper panel, the plots for total directional connectedness "from" others are in the middle panel, and the plots for "net" total directional connectedness to others are in the lower panel.

One of the first things one notices in Fig. 3 is the substantial difference between the "to" and "from" connectedness plots: the "from" connectedness plots are much smoother compared to the "to" connectedness plots. The difference between the two directional connectedness measures is not hard to explain. When there is a shock to the return volatility of an individual stock or a couple of stocks, this volatility shock is expected to be transmitted to other stocks. Since individual institutions' stocks are subject to idiosyncratic shocks, some of these shocks are very small and negligible, while others can be quite large. Irrespective of the size of the volatility shock, if it is the stock of a larger institution or a highly central institution (which has strong balance-sheet and off-balance-sheet connections with other banks) that received the volatility shock, then one can expect this volatility shock to have even a larger spillover effect on stocks of other institutions. As the size of the shocks vary as well as the size and centrality of the institutions in our sample, the directional connectedness "to" others varies substantially across stocks over the rolling-sample windows.

We have already emphasized that the institutions in our sample are the largest ones in the US financial industry. As a result, none of the stocks in our sample of thirteen institutions are insulated from the volatility shocks to other institutions' stocks. In other words, they are expected to be interconnected. As a result, each one will receive, in one form or the other, the volatility shocks transmitted by other institutions. While the volatility shocks transmitted "to" others by each individual stock may be large, when they are distributed among twelve other stocks the size of the volatility shock received by each stock will be much smaller. That is why there is much less variation in the directional connectedness "from" others compared to the directional connectedness "to" others in Fig. 3.

The difference between the directional connectedness "to" and "from" others is equal to the "net" directional connectedness to others presented in the lower panel of Fig. 3. As the connectedness "from" others measure is smoother over the rolling-sample windows, the variation in the plots for "net" connectedness to others over the rolling-sample windows resembles the variation in the plots for connectedness "to" others.

In Fig. 3 we observe that even though for each stock the "from" connectedness reached the highest levels during the 2007–08 crisis, we do not observe such a level shift in the "to" and "net" connectedness measures over the same period. This is so, perhaps because idiosyncratic shocks have always hit individual stocks and these shocks have been transmitted to other stocks. During the

30 The link between the volatility connectedness and the long-term rates is directly a result of the choices of the investors. Rising long-term interest rates reflect optimism about the future economic performance. As they expect the growth to pick up, investors sell more defensive stocks such as the financial stocks and instead invest in manufacturing, energy and airlines sector stocks that are likely to benefit most from an economic recovery.

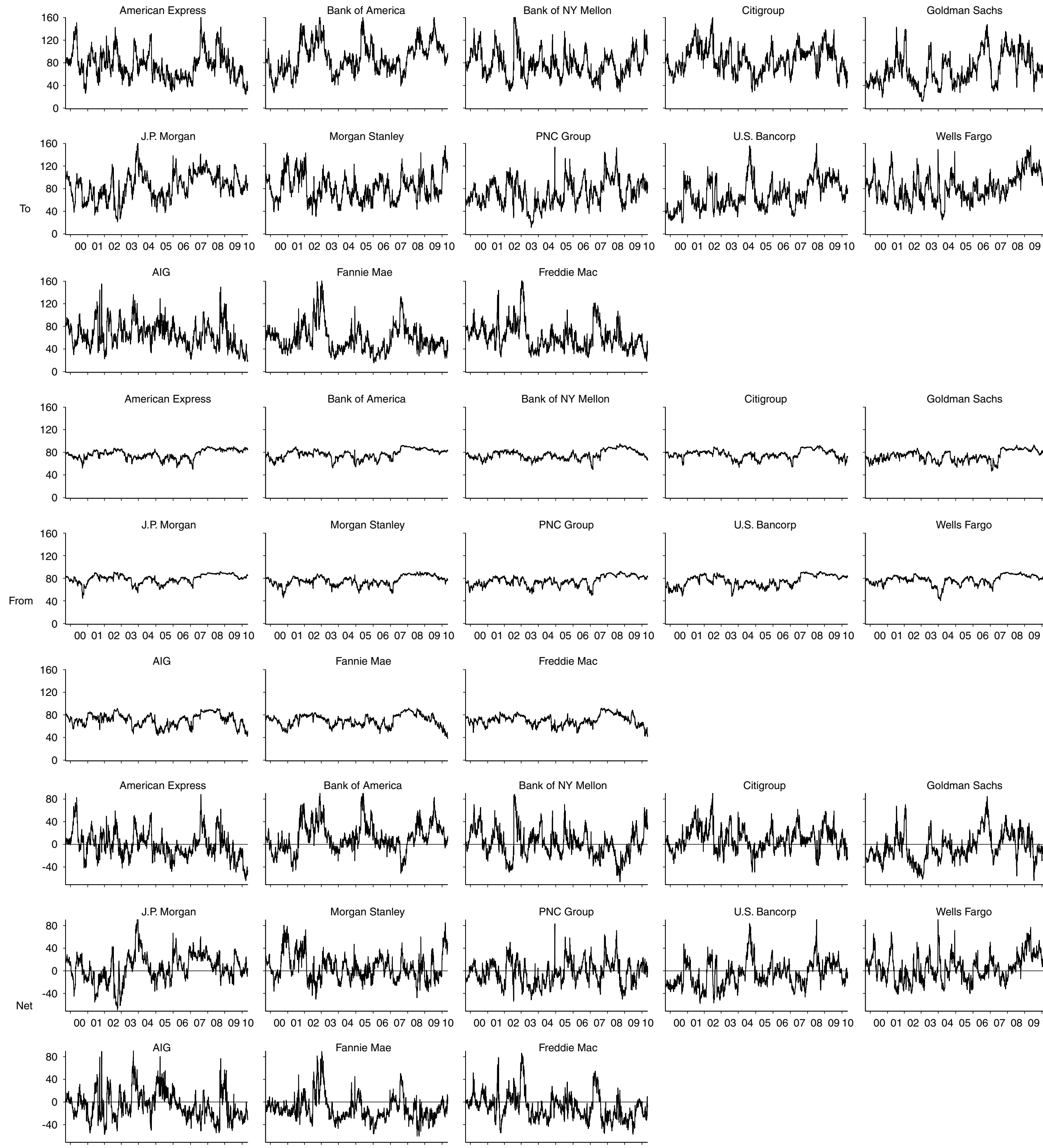


Fig. 3. Rolling Total Directional Connectedness. The rolling estimation window width is 100 days, and the predictive horizon for the underlying variance decomposition is 12 days.

2007–08 crisis these shocks became more frequent and each time hit more stocks than before the crisis and hence were transmitted to others in larger amounts than before.

To better evaluate the differences between the "to" and "from" directional connectedness, in Fig. 4 we plot the evolution of the entire "to" and "from" degree *distributions*. Although, by definition, the *mean* "to" and "from" directional connectedness measures are both equivalent to the total connectedness measure presented in Fig. 2, each financial institution has rather different "to" and "from" directional connectedness. This implies that even though their means are the same, "to" and "from" connectedness measures are distributed quite distinctively. As emphasized earlier, the variation in the "from" connectedness is much lower than the variation in "to" connectedness. Even the first and second quartile band for the "to" connectedness is wider than the min–max range for the "from" connectedness.

Temporal changes in the dispersion and skew of the "to" and "from" connectedness in Fig. 4 may contain useful information. For example, it appears that "from" connectedness gets not only more dispersed but also more left-skewed during crises, and simultaneously that "to" connectedness gets more right-skewed. That is, during crisis times relatively more than non-crisis times,

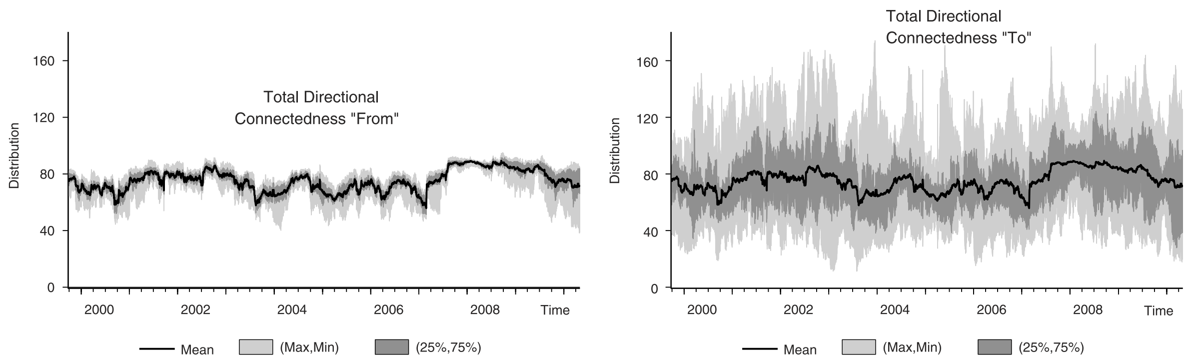


Fig. 4. Rolling Distribution of Total Directional Connectedness. We plot the time series of daily min, 25%, mean, 75%, and max of the distributions of "to" and "from" total directional connectedness. The rolling estimation window width is 100 days, and the predictive horizon for the underlying variance decomposition is 12 days.

there are a few firms receiving very little, and a few firms transmitting very much. One might naturally want to identify firms that are simultaneously "recipients of small" and "transmitters of big" — those are the distressed firms potentially poised to wreak havoc on the system.

5.3.3. Pairwise directional

In the analysis of the full-sample volatility connectedness in Section 5.2, we discussed the importance of pairwise volatility connectedness measures. In particular, we emphasized the importance of pairwise connectedness as a measure of how volatility shocks are transmitted across financial institution stocks. The relevance of the pairwise connectedness measures carries over to the rolling-sample windows. Indeed, the analysis of pairwise connectedness is even more crucial in the rolling-sample windows case, because it helps us identify how the connectedness measures across financial institution stocks vary over time. During times of crises, individual stocks are likely to be subject to frequent volatility shocks. How these shocks led to volatility connectedness across pairs of stocks is very crucial for any analysis of crises. Unfortunately, given that there are 13 institutions in our sample from 1999 to 2010, presenting plots of the volatility connectedness (for each of the 156 pairwise directional measures, and 78 net pairwise directional measures) is an almost impossible task to accomplish in the confines of this paper. Instead, when we are discussing the development of the global financial crisis over time and the volatility connectedness of the most troubled financial institutions during the crisis, we will present and discuss the net pairwise connectedness measures during the most critical days of the crisis.

5.3.4. Robustness assessment

Finally, we conclude this section with a discussion of the robustness of our results to the choice of the parameters of the model. In particular, we plot the total connectedness for two alternative identification methods (namely, the Cholesky factor identification and the generalized identification), for alternative values of the window width (in addition to $w = 100$ days, we consider sample windows of 75 and 125 days), and for alternative forecast horizons (in addition to $H = 12$ days, we consider 6 and 18 days). The results are presented in Fig. 5. In each plot, the solid line is the total connectedness measure obtained through the generalized identification for each value of H and w. In the case of Cholesky factor identification, we calculate the connectedness index for 100 random orderings of the realized stock return volatilities. The gray band in each plot corresponds to the (10%, 90%) interval based on these 100 randomly-selected orderings.

In all subgraphs, the solid line that corresponds to the generalized identification-based total connectedness measure runs higher than the gray band that corresponds to the Cholesky identification. As the generalized identification treats each variable to be ordered as the first variable in the VAR system, the total connectedness obtained from the Cholesky-based identification is the lower bound of the one obtained from the generalized identification. Nevertheless, in all subgraphs of Fig. 5, the two series move very much in accordance over time, a strong indication of the robustness of our total connectedness measures based on generalized identification. It is also important to note that the (10%, 90%) interval based on 100 random orderings of the Cholesky-based total connectedness is quite narrow. The ordering of the financial stocks in the VAR does not really matter much to follow the dynamic behavior of total connectedness.

As the window length, w, is increased, the gap between total connectedness based on the generalized identification and the one based on the Cholesky identification increases. Both connectedness measures are more wiggly when the window width is set to 75 days, but become smoother as we increase the window width to 125 days. Similarly, given the window length, a shorter forecast horizon, H, implies a smaller gap between the generalized- and Cholesky-based total connectedness measures.

To summarize, our robustness checks show that the dynamic behavior of the total connectedness measures over the rolling-sample windows is robust to the choice of alternative sample window lengths, forecast horizons, identification methods and orderings of stocks in the VAR system.

5.4. The Financial Crisis of 2007–2008

Having analyzed the dynamics of the various connectedness measures over time, in this section we focus on the global financial crisis, from 2007 through the end of 2008. The analysis of this section shows how the measurement and daily monitoring of connectedness can help us understand the developments at each stage of the global financial crisis.

5.4.1. Total connectedness at various stages of the crisis

As of the end of 2006 there were already some, albeit weak, signs of slowdown in the US real estate market.[31] In late February 2007, the New Century Financial Corporation was reported to have troubles in servicing its debt. It was followed by the bankruptcy of three small mortgage companies. These in turn worsened the expectations about the real estate markets, the mortgage-backed securities (MBS) markets as well as the stock market, and on the last day of February 2007, the total connectedness measure jumped by more than 17 points, the biggest increase on a single day. The

[31] The Case–Shiller home price index for 20 metropolitan regions was 2% lower in January 2007 compared to its historical high level reached in July 2006.

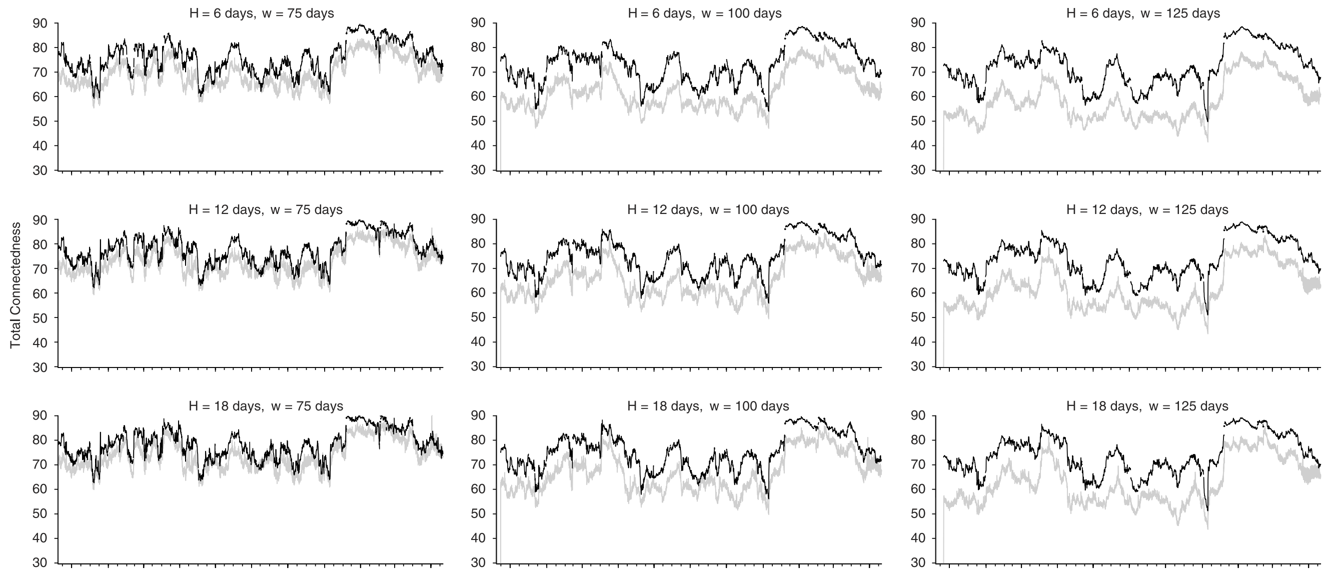


Fig. 5. Robustness of Total Connectedness. We explore estimation window widths w of 75, 100 and 125 days, predictive horizons H of 6, 12 and 18 days, and a variety of Cholesky orderings. In each subgraph, the solid line corresponds to our benchmark ordering, and the gray band corresponds to a [10%, 90%] interval based on 100 randomly-selected orderings.

increase in the total connectedness was not due to a volatility shock to the stock of a single financial institution; rather, all bank stocks were affected by the recent developments in the MBS markets.

The churning in the MBS markets continued from February until early June. New Century declared bankruptcy in April. In June and July the markets became aware that big financial institutions were not insulated from the debacle in the MBS. Bear Stearns had to liquidate two of its hedge funds in July, leading to billions of dollars of losses. From early March to late June the total volatility connectedness index climbed gradually from 73% to 80% (see Fig. 2).

In July 2007, the market for asset-backed commercial paper (ABCP) showed signs of drying up, which eventually led to the liquidity crisis of August 2007. From July 25 to August 10, the index climbed 12 percentage points, to reach 88% (see Fig. 2). Reflecting the developments over the period, the total connectedness index doubled in the first eight months of 2007. After the liquidity crisis of August 2007, it was obvious that the whole financial system would be badly bruised by the collapse of the ABCP market.

After seven months of learning about the problems in MBS markets and the ensuing liquidity crisis, next came the months of reckoning with the consequences as nearly all US banks started to announce huge losses. Even though it had already reached its historical maximum, in late 2007 the volatility connectedness index continued its upward move by several points.

As the MBS markets continued their descent in early 2008 Bear Stearns' financial position became untenable. Amid widespread rumors of an eventual bankruptcy, its stock price declined rapidly in mid-March, briefly increasing the tensions and volatility in the markets. In an operation directed by the New York Fed, J.P. Morgan acquired Bear Stearns on March 17, 2008, with financial assistance from the Fed. As a result of the timely rescue operation, in the final days of Bear Stearns the total connectedness of the surviving thirteen banks showed an upward movement of only a couple of percentage points.

In the summer of 2008 the tension in the stock market had started to build up again as a result of Wachovia Bank's troubles. Thanks to Wachovia's high volatility, the total volatility connectedness index increased, reaching to 88.5% in mid-July (see Fig. 2).

Meanwhile, regional banks smaller than Wachovia failed. These were followed by news about the constantly deteriorating asset positions of Fannie Mae and Freddie Mac. Before going bankrupt, these two "government-sponsored enterprises" were taken into government conservatorship in the first week of September.

Then came the most significant event in the unfolding of the crisis. Following the news that Lehman would announce huge losses in its latest financial statement, market participants started selling Lehman Brothers' stock. Despite the overwhelming efforts over the weekend of September 13–14, no viable takeover bid could be produced for Lehman Brothers by the interested institutions. The US government did not want to step in to save Lehman Brothers with taxpayers' money. As soon as Lehman Brothers declared bankruptcy on the morning of September 15, 2008 all hell broke out in financial markets around the world. That same day, the weakest of the three remaining investment banks, Merrill Lynch, announced it was being acquired by Bank of America. The total volatility connectedness index increased further to reach its maximum level of 89.2% (see Fig. 2).

After months of gyrations in the US financial system, the volatility connectedness started to subside toward the end of the first quarter of 2009. In March and April 2009, the total connectedness measure fluctuated between 80%–85% for a while. It started to fall only after the announcement of the stress test results in May 2009. By October 2009 the index was down to 70%. However, the news coming from Greece and the EU's inability to handle the Greek debt crisis in an orderly manner led to further volatility in financial industry stocks in the EU and the US, which prevented the volatility connectedness index from declining any further. As of the end of our sample, the index was fluctuating between 70% and 75%, a range that is above the levels the index attained during tranquil times (see Fig. 2).

5.4.2. *Pairwise connectedness of troubled financial institutions*

So far we have discussed the behavior of the total connectedness and total directional connectedness measures for a group of thirteen institutions along with the background of the events that took place in the US financial markets during the financial crisis of 2007–2008. Our analysis did not include four major banks that disappeared during the crisis through bankruptcy or acquisitions. In the remainder of this section, we analyze the total directional and pairwise directional connectedness measures for these four institutions as well as for AIG and Morgan Stanley, two other troubled institutions. In Table 4 we list the information on the four major

Table 4
Detail for Financial Institutions Acquired or Bankrupted During the Crisis of 2007–2008. Inv Bank denotes an investment bank, and Com Bank denotes a commercial bank. Market capitalizations are in billions of US dollars.

Institution	Ticker	Business	Market Cap. (12/29/2006)	Important events	
Bear Stearns	BSC	Inv Bank	19	Acquired by JPM	3/17/2008
Lehman Brothers	LEH	Inv Bank	41	Bankruptcy	9/15/2008
Merrill Lynch	MER	Inv Bank	82	Acquired by BAC	9/15/2008
Wachovia Bank	WB	Com Bank	115	Acquired by WFC	10/3/2008

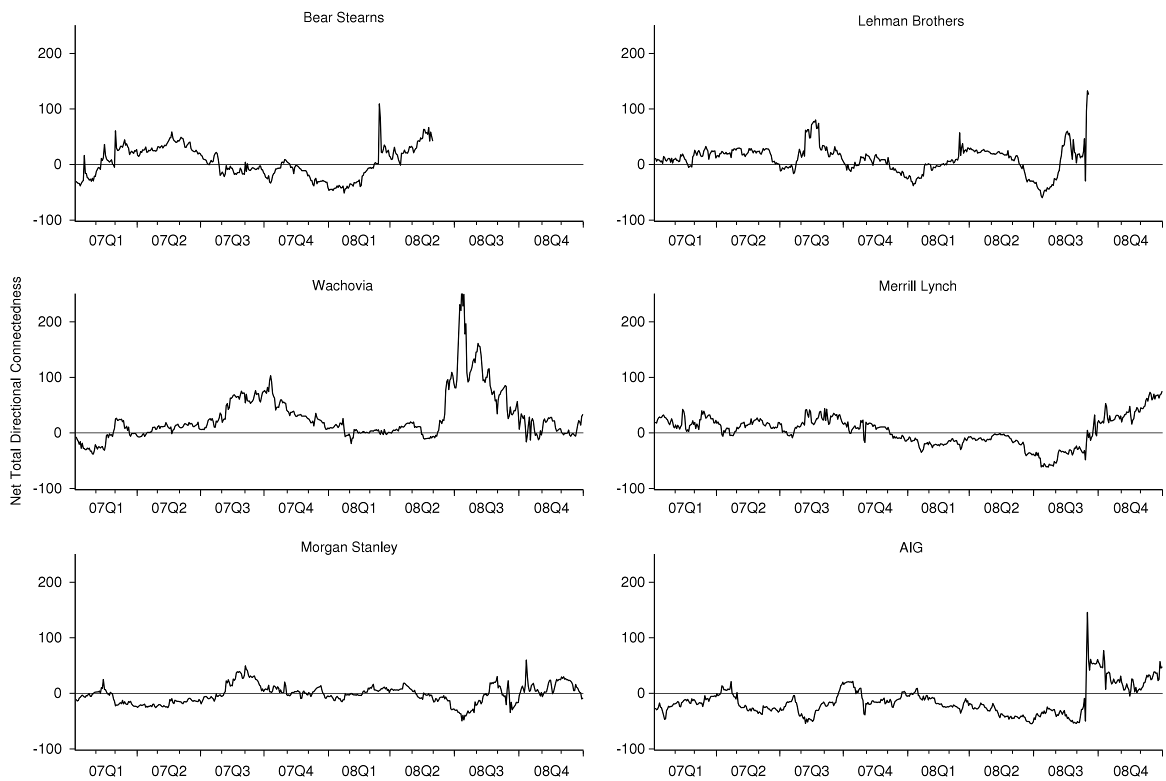


Fig. 6. Net Total Directional Connectedness of Troubled Financial Firms. Net total directional connectedness for firm *i* is "from *i* to others" less "from others to *i*".

banks that ceased to exist, with information on their stock tickers, market capitalization before and after the crisis, and critical dates during the crisis.

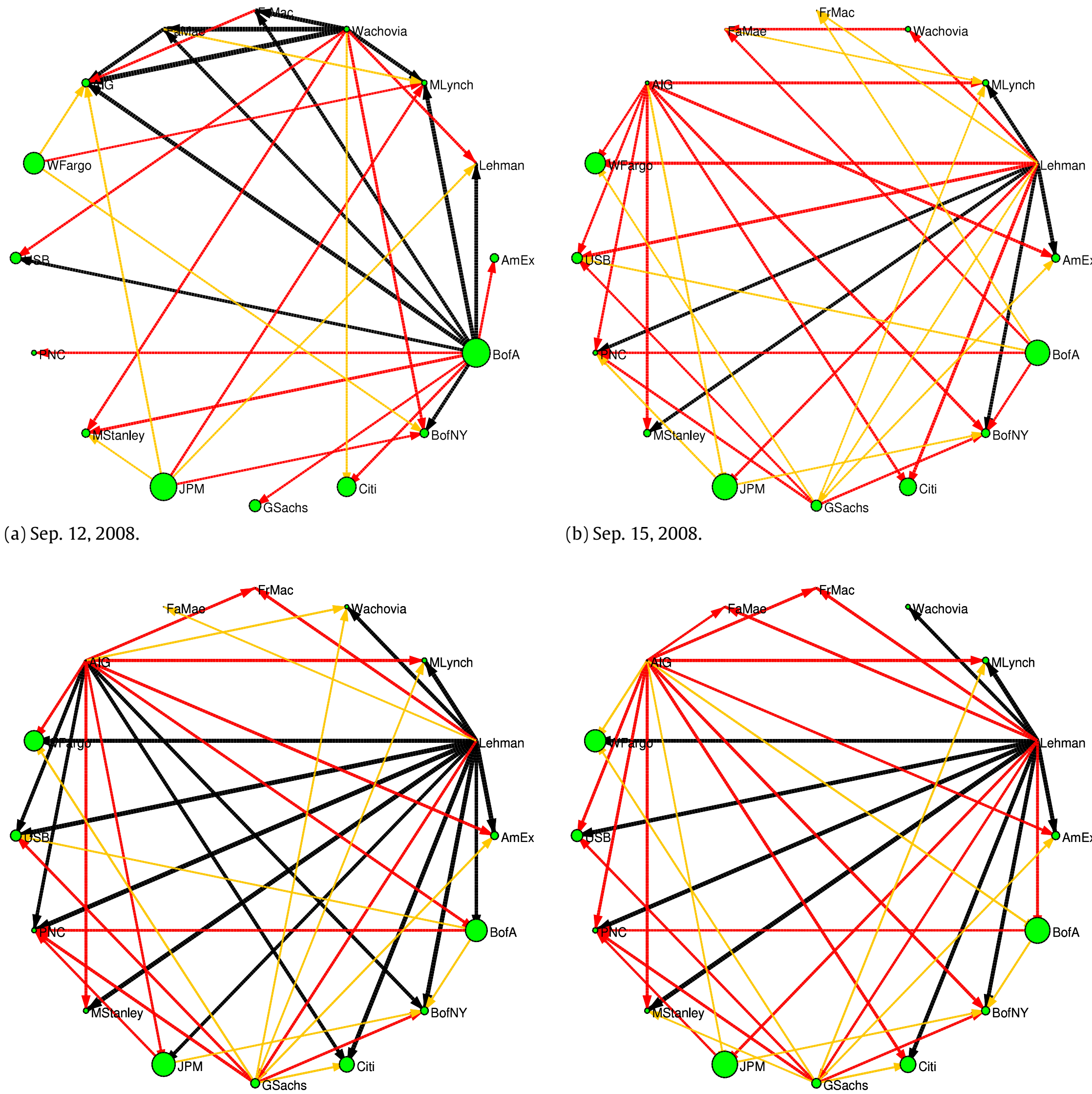
We present net total directional connectedness plots for AIG, Wachovia, Merrill Lynch, Lehman Brothers, Morgan Stanley and Bear Stearns in Fig. 6.[32] Let us spell out the most important observation in Fig. 6 upfront: even though it was the troubles of the investment banks that were followed the most throughout the crisis, Wachovia Bank is the one that had the highest net total and pairwise volatility connectedness in the climactic months of the second half of 2008.

Coming back to the four troubled investment banks, it was true that they had high net connectedness on several occasions as the global financial crisis unfolded steadily in 2007 and 2008. To start with the most vulnerable of the top five investment banks, the net volatility connectedness of Bear Stearns' stock was not sizable in the run-up to its takeover by J.P. Morgan on March 17, 2008, but it increased substantially to 109% on March 14 and 83% on March 17 (see Fig. 6).

Viewed as the most vulnerable investment bank after Bear Stearns, Lehman Brothers' net directional connectedness during the liquidity crisis of August 2007 reached 80%. It also generated close to 57% net directional connectedness on the day Bear Stearns was taken over by J.P. Morgan (Fig. 6). Furthermore, its net directional connectedness stayed around 20% for almost three months after the demise of Bear Stearns. From early June till early August 2008 Lehman Brothers stayed as a net receiver of volatility shocks. This status, however, did not last for long. Lehman again became one of the front runners in terms of net directional connectedness (close to 60%) in the first 20 days of August.

On Friday, September 12, 2008, just one day before the critical weekend, Lehman Brothers was not at the center stage in terms of volatility connectedness; its net total directional volatility connectedness was less than 20% (see Fig. 6). Markets were still expecting another government-orchestrated rescue operation. Only after the announcement of its bankruptcy on the morning of September 15 did Lehman Brothers' stock move to center stage in the crisis and generated substantial volatility connectedness, with a net total directional connectedness of 96% (see Fig. 6). Its net pairwise connectedness with five financial stocks was in the top percentile (another five were in the top five percentiles and two in the top ten percentiles) of all the net pairwise volatility

[32] It is worth noting that connectedness measurements generally will not, and should not, be robust to the choice of reference universe. Hence, given a decision as to the *x* to be examined, a second important issue is precisely which (and hence how many) *x*'s to use. For example, in this paper's analysis of individual financial institution equity return volatilities, we intentionally use only the largest firms. In addition, note that our reference universe will change with the "births" and "deaths" of financial firms. Births happen, for example, when a firm goes public, as with Goldman Sachs in 1999, and deaths happen when firms go bankrupt, as with Lehman Brothers in 2008.



(a) Sep. 12, 2008.
(b) Sep. 15, 2008.
(c) Sep. 16, 2008.
(d) Sep. 17, 2008.

Fig. 7. Net Pairwise Directional Connectedness During the Lehman Bankruptcy. Notes: we show the most important directional connections among the pairs of sixteen bank stocks on each day. Black, red and orange links (black, gray and light gray when viewed in grayscale) correspond to the first, fifth and tenth percentiles of all net pairwise directional connections from June 1 to December 31, 2008. Node size indicates stock market capitalization. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

connectedness that took place between June 1 and December 31 of 2008 (Fig. 7(b)). Lehman Brothers' net pairwise directional connectedness increased substantially in the last two trading days of the stock, September 16 and 17 (see Fig. 7(c) and (d)).

As we have already emphasized above in Fig. 6, among the six troubled banks Wachovia was the one that had the highest net directional connectedness with other stocks. Wachovia's problems had already been known in 2007, yet the markets learned that they were actually worse than previously known when the bank announced that it incurred a loss of \$8.9 billion in the second quarter of 2008. In the month of June, long before the climax month (September 15–October 15) of the financial crisis, Wachovia's stock came under heavy pressure and its net directional connectedness (see Fig. 6) increased substantially to reach 250% in mid-July. On October 3, Wachovia was sold to Wells Fargo.

6. Concluding remarks

Schweitzer et al. (2009) provide an insightful description of the challenges of financial network modeling:

> "In the complex-network context, 'links' are not binary (existing or not existing), but are weighted according to the economic interaction under consideration.... Furthermore, links represent traded volumes, invested capital, and so on, and their weight can change over time". [p. 423]

We hope to have successfully confronted the issues raised by Schweitzer et al., proposing connectedness measures at all levels – from system-wide to pairwise – that are rigorous in theory and readily implemented in practice, that capture the different strengths of different connections, and that capture time-variation in connectedness. Our approach effectively marries VAR variance-decomposition theory and network topology theory, recognizing that variance decompositions of VARs form weighted directed networks, characterizing connectedness in those networks, and in turn characterizing connectedness in the VAR.

We have emphasized the usefulness of "connectedness thinking" for risk measurement and management, but it has risk measurement/management uses beyond those that we emphasized. For example, because connectedness is linked to MES and CoVar, it is implicitly also linked to the idea of stress testing, because the conditioning in MES and CoVaR amounts to conditioning on stress scenarios. Moreover, connectedness may find many other uses, in areas as seemingly diverse as asset pricing (Which risks are truly systematic and hence should be priced?), portfolio management (How best to assess and manage portfolio concentration/diversification dynamically), and policy (Which banks to bail out? Which mergers to approve?).[33] Moreover, much remains to be done even within the confines of risk measurement, not least relating our connectedness measures to others, some of which have received attention such as the equi-correlation measure of Engle and Kelly (2012), and some of which have not yet received attention but appear quite natural, such as the (time-varying) fraction of variation explained by the first principal component.

We see our paper as part of a vibrant emergent literature using network perspectives in economic contexts, and introducing economic perspectives in network contexts. Leading examples include Acemoglu et al. (2010); Adamic et al. (2010); Allen et al. (2012), and Billio et al. (2012). Indeed Billio et al. (2012) is quite a close relative, using pairwise Granger-causality to characterize network structure. The Granger-causal approach is in some respects less appealing than ours (e.g., it is directional but exclusively pairwise and unweighted, testing zero vs. nonzero coefficients, with arbitrary significance levels, and without tracking the magnitude of non-zero coefficients), and in other respects more appealing (e.g., there is no need for identifying assumptions, which are inescapable in variance-decomposition and impulse-response analyses), and the two are surely complements rather than substitutes. In any event it seems clear that the network and multivariate time series literatures have much to learn from each other, and that their blending may have much to contribute to the successful measurement of financial economic risks.

Acknowledgments

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[33] Indeed the IMF is now actively using our connectedness measures; see, for example, its Global Financial Stability Report at http://www.imf.org/external/pubs/ft/gfsr/2011/02/pdf/text.pdf.