A New Index of Financial Conditions*

Gary Koop
University of Strathclyde

Dimitris Korobilis†
University of Glasgow

December 1, 2013

Abstract

We use factor augmented vector autoregressive models with time-varying coefficients and stochastic volatility, to construct a financial conditions index that can accurately track expectations about growth in US GDP and unemployment. Time-variation in the model's parameters allows for the weights attached to each financial variable in the index to evolve over time. Furthermore, we develop methods for dynamic model averaging or selection which allow the financial variables entering into the FCI to change over time. We discuss why such extensions of the existing literature are important and show them to be so in an empirical application involving a wide range of financial variables.

Keywords: Bayesian model averaging; dynamic factor model; dual Kalman filter; forecasting

JEL Classification: C11, C32, C52, C53, C66

*We want to thank Gianni Amisano, Sylvia Kaufmann, Michael McCracken, and Francesco Ravazzolo for helpful comments. Special acknowledgements to Barbara Rossi for her extensive and insightful suggestions. Finally we want to thank participants at: the 21th Symposium of the Society for Nonlinear Dynamics and Econometrics in Milan; the Conference Forecasting Structure and Time Varying Patterns in Economics and Finance in Rotterdam; the 7th International Conference on Computational and Financial Econometrics in London, and seminar attendees at Birkbeck College, Norges Bank, and Univesité de Rennes 1.
This research was supported by the ESRC under grant RES-062-23-2646.

†Corresponding author. Address: University of Glasgow, Adam Smith Business School, Gilbert Scott building, Glasgow, G12 8QQ, United Kingdom, Tel: +44 (0)141 330 2950, e-mail: Dimitris.Korobilis@glasgow.ac.uk

1 Introduction

The recent financial crisis has sparked an interest in the accurate measurement of financial shocks to the real economy. An important lesson of recent events is that financial developments, not necessarily driven by monetary policy actions or fundamentals, may have a strong impact on the economy. The need for policy-makers to closely monitor financial conditions is clear. In response to this need, a recent literature has developed several empirical econometric methods for constructing financial conditions indexes (FCIs). These indexes contain information from many financial variables, and the aim is for policy-makers to use them to provide early warning of future financial crises. As a result, many financial institutions (e.g. Goldman Sachs, Deutsche Bank and Bloomberg) and policy-makers (e.g. the Federal Reserve Bank of Kansas City) produce closely-watched FCIs. Estimation of such FCIs ranges from using simple weighted averages of financial variables through more sophisticated econometric techniques. An important recent contribution is Hatzius, Hooper, Mishkin, Schoenholtz and Watson (2010) which surveys and compares a variety of different approaches. The FCI these authors propose is based on simple principal components analysis of a very large number of quarterly financial variables. Other recent notable studies in this literature include English, Tsatsaronis and Zoli (2005), Balakrishnan, Danninger, Elekdag and Tytell (2008), Beaton, Lalonde and Luu (2009), Brave and Butters (2011), Gomez, Murcia and Zamudio (2011) and Matheson (2011).

In this paper our goal is to accurately monitoring financial conditions through a single latent (unobserved) FCI. We argue that the construction and use of an FCI involves three issues: i) selection of financial variables to enter into the FCI, ii) the weights used to average these financial variables into an index and iii) the relationship between the FCI and the macroeconomy. There is good reason for thinking all of these may be changing over time. Indeed, Hatzius et al (2010) discuss at length why such change might be occurring and document statistical instability in their results. For instance, the role of the sub-prime housing market in the financial crisis provides a clear reason for the increasing importance of variables reflecting the housing market in an FCI. A myriad of other changes may also impact on the way an FCI is constructed, including the change in structure of the financial industry (e.g. the growth of the shadow banking system), changes in the response of financial variables to changes in monetary policy (e.g. monetary policy works differently with interest rates near the zero bound), the changing impact of financial variables on real activity (e.g. the role of financial variables in the recent recession is commonly considered to have been larger than in other recessions) and several other examples.

Despite such concerns about time-variation, the existing literature does little to statistically model it. Constant coefficient models are used with, at most, rolling methods to account for time-variation. Furthermore, many FCI's are estimated ex post, using the entire data set. So, for instance, at the time of the financial crisis, some FCIs will be based on financial variables which are selected after observing the financial crisis and the econometric model will be estimated using financial crisis data. The major empirical contribution of the present paper is to develop an econometric approach which allows for different financial variables to affect estimation of the FCI, with varying (or zero, when not selected) weight each. That way, we develop an econometric tool that explicitly takes into account the fact that each financial crisis has different causes, and is transmitted to the real economy with varying intensity.

Following a common practice in constructing indexes, we use factor methods. To be precise, we use Factor-augmented VARs (FAVARs) which jointly model a large number of financial variables (used to construct the latent FCI) with key macroeconomic variables. Following the recent trend in macroeconomic modelling using VARs and FAVARs (Primiceri, 2005; Korobilis, 2013) we work with time-varying parameter FAVARs (TVP-FAVARs) which allow coefficients and loadings to change in each period. Additionally, we work with a large set of (TVP-) FAVARs that differ in which financial variables are included in the estimation of the FCI. Faced with a large model space and the desire to allow for model change, we follow Koop and Korobilis (2012) and use efficient methods for Dynamic Model Selection (DMS) and Dynamic Model Averaging (DMA). These methods forecast at each point in time with a single optimal model (DMS), or reduce the expected risk of the final forecast by averaging over all possible model specifications (DMA). We implement model selection or model averaging in a dynamic manner. That is, DMS chooses different financial variables to make up the FCI at different points in time. DMA constructs an FCI by averaging over many individual FCIs constructed using different financial variables. The weights in this average vary over time.

From an econometrician's point of view, there is also growing theoretical evidence in favor of our modelling strategy. Boivin and Ng (2006) show that using all available data to extract factors (the FCI in our case) is not always optimal in factor analysis, thus providing support for implementing DMA/DMS to construct our FCI. Additionally, there is much econometric evidence in favor of structural instabilities in the coefficients or loadings of macroeconomic and financial factor models; see, among others, Banerjee, Marcellino and Masten (2006) and Bates, Plagborg-Møller, Stock and Watson (2013).

Econometric methods for estimating FAVARs and TVP-FAVARs are well-established; see, e.g., Bernanke, Boivin and Eliasz (2005), and Korobilis (2013). However, typical likelihood-based estimation techniques used in the literature (e.g. Bayesian methods using Markov chain Monte Carlo algorithms) rely on simulation algorithms or complex numerical methods, all of which are computationally extremely demanding in high dimensions. With our large model space, and our wish to implement recursive forecasting, it is computationally infeasible to use such methods. Therefore, our major econometric contribution in this paper lies in the development of fast estimation methods which are based on the Kalman filter and smoother and are simulation-free. When dealing with the FAVAR with constant parameters, our algorithm collapses to the two-step estimator for dynamic factor models of Doz, Giannone and Reichlin (2011). In the case of estimating models with time-varying parameters and stochastic volatility (TVP-FAVARs), our algorithm provides an extension of Doz, Giannone and Reichlin (2011).

Our results indicate that financial variables do have predictive power for measures of output growth (GDP and unemployment). Additionally, time variation in the parameters is extremely important for providing accurate short-run forecasts. Finally, model averaging and/or selection also result in improvement of forecast accuracy over using a single model with all the available financial variables. In the remainder of the paper we examine all these issues in depth, and we provide evidence by using different forecast metrics and by conducting several robustness checks.

In particular, in the next Section we introduce formally our modeling framework and sketch the features of our novel estimation algorithm (complete details are provided in the Technical Appendix), plus we describe how we implement DMA or DMS methods in the face of the large number of models we work with. In Section 3 we present our data, estimates of different FCIs, and results of a recursive forecasting exercise which is the main tool for evaluating the conditions under which we can obtain an optimal FCI. Section 4 concludes the paper.

2 Factor Augmented VARs with Structural Instabilities

2.1 The TVP-FAVAR Model and its Variants

Let x_t (for $t = 1, ..., T$) be an $n \times 1$ vector of financial variables to be used in constructing the FCI. Let y_t be an $s \times 1$ vector of macroeconomic variables of interest. In our empirical work $y_t = (\pi_t, g_t, u_t, m_t, r_t)'$ where π_t is the CPI inflation rate, g_t is the growth rate of GDP, u_t is the unemployment rate, m_t is the growth rate of money supply, and r_t is the interest rate[1]. The p-lag TVP-FAVAR takes the form

$$\begin{gathered} x_t = \lambda_t^y y_t + \lambda_t^f f_t + u_t \\ \begin{bmatrix} y_t \\ f_t \end{bmatrix} = c_t + B_{t,1} \begin{bmatrix} y_{t-1} \\ f_{t-1} \end{bmatrix} + ... + B_{t,p} \begin{bmatrix} y_{t-p} \\ f_{t-p} \end{bmatrix} + \varepsilon_t \end{gathered} , \tag{1}$$

[1]Note that throughout this paper past data up to time t will be denoted by $1:t$ subscripts, e.g., $Data_{1:t} = (Data_1, .., Data_t)$. Estimates of time varying parameters or latent states can be made using data available at time $t-1$ (filtering), or time t (updating) or time T (smoothing). We use subscript notation for this such that $a_{t|\tau}$ is an estimate (or posterior moment) of time-varying parameter a_t using data available through period τ.

where λ_t^y are regression coefficients, λ_t^f are factor loadings, f_t is the latent factor which we interpret as the FCI, c_t is the intercept and $(B_{t,1}, ..., B_{t,p})$ are VAR coefficients. u_t and ε_t are zero-mean Gaussian disturbances with time-varying covariances V_t and Q_t, respectively. We adapt the common identifying assumption in the factor literature that V_t is diagonal, thus ensuring that u_t is a vector of idiosyncratic shocks and f_t contains information common to the financial variables. This model is very flexible since it allows all parameters to take a different value at each time t. Such an assumption is important since there is good reason to believe that there is time variation in the loadings and covariances of factor models which use both financial and macroeconomic data (see Banerjee, Marcellino and Masten, 2006). For recent discussions about the implication of the presence of structural breaks in factor loadings, the reader is referred to Breitung and Eickmeier (2011) and Bates, Plagborg-Møller, Stock and Watson (2013).

Following the influential work of Bernanke, Boivin and Eliasz (2005) our factor model in (1) consists of two sub-equations: one equation which allows us to extract the latent financial conditions index (FCI) from financial variables x_t; and one equation which allows to model the dynamic interactions of the FCI with macroeconomic variables y_t. This econometric specification is important for two reasons. First, unlike Stock and Watson (2002) who extract a factor and then use it in a separate univariate forecasting regression, we use a multivariate system to forecast macroeconomic variables using the FCI. Thus, we jointly model all the variables in the system which should allow us to better characterize their co-movements and interdependence Second, following the recommendations of Hatzius et al. (2010), we are able to purge from the FCI the effect of macroeconomic conditions. Thus, the final estimated FCI reflects information solely associated with the financial sector.

In order to complete our model, we need to define how the time varying parameters evolve. While the specification of all time-varying covariances is discussed in the following subsection, we define here the vectors of loadings $\lambda_t = \left((\lambda_t^y)', \left(\lambda_t^f\right)' \right)'$ and VAR coefficients $\beta_t = \left(c_t', vec\left(B_{t,p}\right)', ..., vec\left(B_{t,p}\right)'\right)'$ to evolve as multivariate random walks of the form

$$\begin{aligned} \lambda_t &= \lambda_{t-1} + v_t, \\ \beta_t &= \beta_{t-1} + \eta_t, \end{aligned} \tag{2}$$

where $v_t \sim N\left(0, W_t\right)$ and $\eta_t \sim N\left(0, R_t\right)$. Finally, all disturbance terms presented in the equations above are uncorrelated over time and with each other.

We call the full model described in equations (1) and (2) the TVP-FAVAR. We also consider several restrictions on the TVP-FAVAR which result in other popular multivariate models:

1. Factor-augmented time-varying parameter VAR (FA-TVP-VAR): This specification is obtained from the TVP-FAVAR model under the restriction that the

loadings are constant ($W_t = 0$ for all t, in which case $\lambda_t = \lambda_0$). In this case the first equation in (1) describes a typical factor model, while the second equation is a TVP-VAR augmented with the FCI.

2. Factor-augmented VAR (FAVAR): This model is obtained from the TVP-FAVAR under the restriction that both λ_t and β_t are time-invariant ($W_t = R_t = 0$).

3. Time-varying parameter VAR (TVP-VAR): This model can be obtained from the TVP-FAVAR under the restriction that the number of factors is zero (i.e. $f_t = 0$).

4. VAR: This model is obtained when the number of factors is zero and both λ_t and β_t are constant over time.

Note that all of the specifications above have heteroskedastic covariances V_t and Q_t. We have also worked with homoskedastic versions of the above models (i.e. where V_t and Q_t are constant over time). However, in line with the recent VAR literature (e.g. Clark, 2009), we have found that homoskedastic models are always dominated in forecasting by their heteroskedastic counterparts. Consequently, all the results presented in this paper use heteroskedastic models.

2.2 Estimation of a Single TVP-FAVAR

Bayesian estimation of FAVARs (as well as VARs) with time-varying parameters is typically implemented using Markov Chain Monte Carlo (MCMC) methods, which sample from the very complex (nonlinear) and multivariate joint posterior density of the factor f_t and the remaining model parameters; see, e.g., Primiceri (2005), or Del Negro and Otrok (2008). Such Bayesian simulation methods are computationally expensive even in the case of estimating a single TVP-FAVAR. When faced with multiple TVP-FAVARs and when doing recursive forecasting (which requires repeatedly doing MCMC on an expanding window of data), the use of MCMC methods is prohibitive.[2]

In this paper, we use a fast two-step estimation algorithm which vastly reduces the computational burden, and greatly simplifies the estimation of the FCI. Following Koop and Korobilis (2013) we combine the ideas of variance discounting methods with the Kalman filter in order to obtain analytical results for the posteriors of the state variable (f_t) as well as the time-varying parameters $\theta_t = (\lambda_t, \beta_t)$. To motivate our methods, note first that, as long as both the factor, f_t, and the

[2]To provide the reader with an idea of approximate computer time, consider the three variable TVP-VAR of Primiceri (2005). Taking 10,000 MCMC draws (which may not be enough to ensure convergence of the algorithm) takes approximately 1 hour on a good personal computer. Thus, forecasting at 100 points in time takes roughly 100 hours. These numbers hold for a single small TVP-VAR, and would be much infeasible for the millions of larger TVP-FAVARs we estimate in this paper.

loadings parameters, λ_t, in the measurement equation are unobserved, application of the typical Kalman filter recursions for state-space models is not possible. Therefore, we adapt ideas from Doz, Giannone and Reichlin (2011) and the state-space literature (Nelson and Stear, 1976) and develop a dual, conditionally linear filtering/smoothing algorithm which allows us to estimate the unobserved state f_t and the parameters $\theta_t = (\lambda_t, \beta_t)$ in a fraction of a second.

The idea of using a dual linear Kalman filter is very simple: first update the parameters θ_t given an estimate of f_t, and subsequently update the factor f_t given the estimate of θ_t. Such conditioning allows us to use two distinct linear Kalman filters or smoothers,[3] one for θ_t and one for f_t. The main approximation involved is that $\widetilde{f}_t$, the principal components estimate of f_t based on $x_{1:t}$, is used in the estimation of θ_t. Such an approach will work best if the principal component(s) provide a good approximation of the factor(s) coming from a FAVAR with structural instabilities. A theoretical proof that this is the case is not available for our flexible and highly nonlinear specification. However, given the recent findings of Stock and Watson (2009) and Bates, Plagborg-Møller, Stock and Watson (2013), there is strong empirical evidence to believe that this is the case. In particular Bates, Plagborg-Møller, Stock and Watson (2013) conduct extensive Monte Carlo experiments and show that principal components can support large amount of time variation in the loading coefficients λ_t.

Error covariance matrices in the multivariate time series models used with macroeconomic data are usually modeled using multivariate stochastic volatility models (Primiceri, 2005), estimation of which also requires computationally intensive methods. In order to avoid this computational burden, we estimate (V_t, Q_t, W_t, R_t) recursively using simulation-free variance matrix discounting methods (e.g. Quintana and West, 1988). The Technical Appendix provides complete details. For V_t and Q_t we use exponentially weighted moving average (EWMA) estimators. These depend on decay factors κ_1 and κ_2, respectively. Such recursive estimators are trivial computationally. Additionally, the EWMA is an accurate approximation to an integrated GARCH model. Such a feature is in line with authors such as Primiceri (2005) and Cogley and Sargent (2005) who, in the context of macroeconomic VARs, work with integrated stochastic volatility models. The covariance matrices W_t, R_t are estimated using the forgetting factor methods described in Koop and Korobilis (2012, 2013)[4] which depend on forgetting factors κ_3 and κ_4, respectively. Decay and forgetting factors have very similar interpretations. Lower values of the decay/forgetting factors imply that the more recent observation $t-1$, and its squared residual, take higher weight in estimating

[3]The other alternative being to use a joint nonlinear filter, e.g. the Unscented Kalman Filter (UKF) and the Extended Kalman Filter (EKF). We have found such filters to be very unstable given the dimension of our model, and the relatively few time-series observations.

[4]An EWMA estimation scheme can also be applied to these matrices, but due to their large dimension we found better numerical stability and precision when using forgetting factors.

V_t and Q_t compared to older observations. The EWMA method implies that an effective window of $\kappa_1/2-1$ $(\kappa_2/2-1)$ observations is used to estimate V_t (Q_t), while the forgetting factor approach implies that an effective window of $1/(1-\kappa_3)$ $(1/(1-\kappa_4))$ observations is used to estimate W_t (R_t). The choice of the decay/forgetting factors can be made based on the expected amount of time-variation in the parameters.[5] Note that the choice $\kappa_1=\kappa_2=1$ make V_t and Q_t constant, while $\kappa_3=\kappa_4=1$ imply that $W_t=R_t=0$ in which case λ_t and β_t are constant.

A simplified version of our estimation algorithm is given in the following

Algorithm for estimation of the TVP-FAVAR

1. a) Initialize all parameters, $\lambda_0, \beta_0, f_0, V_0, Q_0$
   b) Obtain the principal components estimates of the factors, $\widetilde{f}_t$
2. Estimate the time varying parameters θ_t given $\widetilde{f}_t$
   a) Estimate V_t, Q_t, R_t, and W_t using VD
   b) Estimate λ_t and β_t, given (V_t, Q_t, R_t, W_t), using the KFS
3. Estimate the factors f_t given θ_t using the KFS

where VD stands for "Variance Discounting" and KFS stands for "Kalman filter and smoother". The steps above can also be considered to be a generalization of the estimation steps introduced by Doz et al. (2011) for the estimation of constant parameter dynamic factor models. In fact, if we fix all time-varying coefficients and covariances to be constant, our algorithm collapses to the FAVAR equivalent of the two-step estimation algorithm for dynamic factor models of Doz et. al (2011).

Identification in the FAVAR is achieved in a standard fashion by restricting V_t to be a diagonal matrix. This restriction ensures that the factors, f_t, capture movements that are common to the financial variables, x_t, after removing the effect of current macroeconomic conditions through inclusion of the $\lambda_t^y y_t$ term. Further restrictions usually imposed in likelihood-based estimation of factor models, e.g. normalizing the first element of the loadings matrix to be 1 (Bernanke, Boivin and Eliasz, 2005) are not needed here since the loadings λ_t are identified (up to a sign rotation) from the principal components estimate of the factor.

2.3 Dynamic Model Averaging and Selection with many TVP-FAVARs

In this paper, we work with M_j, $j=1,..,J$, models which differ in the financial variables which enter the FCI. In other words, a specific model is obtained using

[5] Choice of forgetting factors is similar in spirit to choice of prior. Empirical macroeconomists frequently impose subjective priors on the degree of time variation in their parameters; see for instance the very informative priors used in the TVP-VARs of Primiceri (2005) and Cogley and Sargent (2005).

the restriction that a specific combination of financial variables have zero loading on the factor at time t or, equivalently, that different combinations of columns of x_t are set to zero. Thus, M_j can be written as

$$\begin{array}{l} x_t^{(j)} = \lambda_t^y y_t + \lambda_t^f f_t^{(j)} + u_t \\ \left[\begin{array}{c} y_t \\ f_t^{(j)} \end{array} \right] = c_t + B_{t,1} \left[\begin{array}{c} y_{t-1} \\ f_{t-1}^{(j)} \end{array} \right] + ... + B_{t,p} \left[\begin{array}{c} y_{t-p} \\ f_{t-p}^{(j)} \end{array} \right] + \varepsilon_t \end{array}, \tag{3}$$

where $x_t^{(j)}$ is a subset of x_t, and $f_t^{(j)}$ is the FCI implied by model M_j. Since x_t is of length n, there is a maximum of $2^n - 1$ combinations[6] of financial variables that can be used to extract the FCI.

When faced with multiple models, it is common to use model selection or model averaging techniques. However, in the present context we wish such techniques to be dynamic. That is, in a model selection exercise, we want to allow for the selected model to change over time, thus doing DMS. In a model averaging exercise, we want to allow for the weights used in the averaging process to change over time, thus leading to DMA. In this paper, we do DMA and DMS using an approach developed in Raftery et al (2010) in an application involving many TVP regression models. The reader is referred to Raftery et al (2010) for a complete derivation and motivation of DMA. Here we provide a general description of what it does.

The goal is to calculate $\pi_{t|t-1,j}$ which is the probability that model j applies at time t, given information through time $t-1$. Once $\pi_{t|t-1,j}$ for $j = 1, .., J$ are obtained they can either be used to do model averaging or model selection. DMS arises if, at each point in time, the model with the highest value for $\pi_{t|t-1,j}$ is used. Note that $\pi_{t|t-1,j}$ will vary over time and, hence, the selected model can switch over time. DMA arises if model averaging is done in period t using $\pi_{t|t-1,j}$ for $j = 1, .., J$ as weights. The contribution of Raftery et al (2010) is to develop a fast recursive algorithm for calculating $\pi_{t|t-1,j}$.

Given an initial condition, $\pi_{0|0,j}$ for $j = 1., , .J$, Raftery et al (2010) derive a model prediction equation using a forgetting factor α:

$$\pi_{t|t-1,j} = \frac{\pi_{t-1|t-1,j}^{\alpha}}{\sum_{l=1}^{J} \pi_{t-1|t-1,l}^{\alpha}}, \tag{4}$$

and a model updating equation of:

$$\pi_{t|t,j} = \frac{\pi_{t|t-1,j} f_j \left(Data_t | Data_{1:t-1} \right)}{\sum_{l=1}^{J} \pi_{t|t-1,l} f_l \left(Data_t | Data_{1:t-1} \right)}, \tag{5}$$

where $f_j \left(Data_t | Data_{1:t-1} \right)$ is a measure of fit for model j. Many possible measures of fit can be used. Since our focus is on the ability of the FCI to forecast y_t, we

[6] We remove from the model set the model with zero financial variables, i.e. with no FCI extracted.

set as a measure of fit the predictive likelihood for the macroeconomic variables, $p_j\left(y_t|Data_{1:t-1}\right)$.

The factor $0 < \alpha \leq 1$ is a forgetting factor which, similar to the decay/forgetting factors $(\kappa_1, \kappa_2, \kappa_3, \kappa_4)$ used for estimating the error covariance matrices, tunes how rapidly switches between models should occur. Lower values of α allow for an increasing switching between the number of variables that enter the FCI each time period. If $\alpha = 0.99$, forecast performance five years ago receives 80% as much weight as forecast performance last period whereas $\alpha = 1$ leads to conventional Bayesian model averaging implemented one period at a time on an expanding window of data.

3 Empirics

3.1 Data and Model Settings

We use 20 financial variables which cover a wide variety of financial considerations (e.g. asset prices, volatilities, credit, liquidity, etc.). These are gathered from several sources. All of the variables (i.e. both macroeconomic and financial variables) are transformed to stationarity following Hatzius et al (2010) and many others. The Data Appendix provides precise definitions, acronyms, data sources, sample spans and details about the transformations. Our data sample runs from 1959Q1 to 2012Q1. Notice that all of our models use four lags and, hence, the effective estimation sample begins in 1960Q1. The five macroeconomic variables that complete our model are the Consumer Price Index (all items), Gross Domestic Product, unemployment rate, M1 money stock, and the Federal funds rate. All of these series are observed from 1959Q1, are seasonally adjusted, and are provided by Federal Reserve Economic Data (FRED). Macro variables which are not already in rates (CPI, GDP, M1) are converted to growth rates by taking first log-differences.

Some of the financial variables have missing values in that they do not begin until after 1959Q1. In terms of estimation with a single TVP-FAVAR model, such missing values cause no problem since they can easily be handled by the Kalman filter (see the Technical Appendix for more details). However, when we are using multiple models, there is a danger that in a specific model the value of the FCI in a period (e.g. 1959Q1-1982Q1) has to be extracted using financial variables which all have missing values for that same period. In this case estimation is impossible and we introduce a simple restriction to prevent such estimation issues. In each model we always include the S&P500 in the list of financial variables, a variable which is observed since 1959Q1, which means that at a minimum the FCI will be extracted based on this single financial variable. This restriction implies that the S&P500 is not subject to model averaging/selection and we instead perform DMA/DMS using the remaining 19 financial variables. Therefore, we have a model space of

2^{19}=524,288 TVP-FAVARs (as well as 524,288 FA-TVP-VARs, etc.). We remind the reader that a list of the different specifications estimated (and their acronyms) is given at the end of Section 2.1.

To summarize, our models which produce an FCI are the TVP-FAVARs, FA-TVP-VARs, FAVARs. In our forecasting exercise, for the purpose of comparison, we also include some forecasting models which do not produce an FCI. These are the VARs and TVP-VARs. With these model spaces, we investigate the use of DMS, DMA and a strategy of simply using the single model which includes all 20 of the financial variables.

Some authors (e.g. Eickmeier, Lemke and Marcellino, 2011) use existing FCIs (i.e. estimated by others) in the context of a VAR model. In this spirit, we also present results for VARs and TVP-VARs where the factors are not estimated from the factor model equation in (1), rather they are replaced with an existing estimate. To be precise, we use $\left(y_t', \widehat{f}_t\right)'$ as dependent variables for different choices of $\widehat{f}_t$. Table 1 lists these choices from a set of financial conditions indexes and financial stress indexes[7] maintained by Federal Reserve Banks. Again, these models are a restricted special case of our TVP-FAVAR and estimation proceeds accordingly. The error covariance matrix is modelled in the same manner as the FAVAR. We use an acronym for these VARs such that, e.g., VAR (FCI 3), is the VAR involving the five macroeconomic variables and the Chicago Fed National FCI.

Table 1. Financial Conditions and Stress Indexes

Name	Acronym	Source	Sample
St. Louis Financial Stress Index	FCI 1	St Louis Fed	1993Q4 - 2012Q1
Kansas City Fed Financial Stress Index	FCI 2	Kansas Fed	1990Q1 - 2012Q1
Cleveland Fed Financial Stress Index	FCI 3	Cleveland Fed	1991Q3 - 2012Q1
Chicago Fed National FCI	FCI 4	Chicago Fed	1973Q1 - 2012Q1

3.2 Choice of hyperparameters and initial conditions

In this section we outline the setting of various hyperparameters and initial conditions. In order to avoid data-mining issues, that is do choices which work well after observing the results, all our benchmark choices that apply in the next two subsections are fairly non-informative. In the Appendix we also implement a sensitivity analysis by means of eliciting priors based on a training data sample, thus extending the recommendations of Primiceri (2005) to our FAVARs.

The first step is to set the initial conditions for the factor f_t (FCI), the time-varying parameters λ_t, β_t, the time-varying covariances V_t, Q_t, and, for doing DMA

[7]Financial Stress Indexes (FSIs) are usually identical to FCIs, but have opposing signs: a decrease in financial conditions means increased financial stress, and vice-versa.

and DMS, we must specify $\pi_{0|0,j}$, $j = 1, ..., J$. These initial conditions are set to the following (relatively non-informative) values

$$
\begin{aligned}
f_0 &\sim N(0,4), \\
\lambda_0 &\sim N\left(0, 4 \times I_{n(s+1)}\right), \\
\beta_0 &\sim N(0, V_{MIN}) \\
V_0 &\equiv 1 \times I_n, \\
Q_0 &\equiv 1 \times I_{s+1}, \\
\pi_{0|0,j} &= \frac{1}{J}
\end{aligned}
$$

where V_{MIN} is a diagonal covariance matrix which, following the Minnesota prior tradition, penalizes more distant lags and is of the form

$$
V_{MIN} = \begin{cases} 4, & \text{for intercepts} \\ 4/r^2, & \text{for coefficient on lag } r \end{cases}, \tag{6}
$$

where $r = 1, .., p$ denotes the lag number. Note that estimates of W_t and R_t are proportional to the respective state covariance matrices obtained from the Kalman filter, therefore there is no need to initialize these matrices; see the Technical Appendix for more details.

Regarding the decay and forgetting factors we have introduced in our model it is worth noting that we can estimate these from the data. However, computation increases substantially (we need to evaluate or maximize the likelihood for each combination of the various factors) and, as shown in Koop and Korobilis (2013), the existence of value added in forecasting performance from such a procedure is questionable. Given these considerations, we choose to fix the values of the decay/forgetting factors, but investigate sensitivity to their choice in the Appendix.

For the decay factors κ_1, κ_2 which control the variation in the covariance matrices, we fix these to the value 0.96. Such value provides volatility estimates which are quite close to the ones expected by integrated stochastic volatility models that have been used extensively in the Bayesian VAR and FAVAR literature (Primiceri, 2005; Korobilis, 2013). For the forgetting factors κ_3, κ_4, we follow the "business as usual prior" approach of Cogley and Sargent (2005) and assume that changes each period are relatively slow and stable under the random walk specification in equation (2). In order to achieve this slow time variation in the coefficients, we set $\kappa_3, \kappa_4 = 0.99$, a setting we use in all TVP-FAVAR and TVP-VAR specifications. As described in Section 2.2, restricted versions of our general model can be obtained by setting the forgetting factors to one. For instance when $\kappa_3 = 1$ but $\kappa_4 = 0.99$, we obtain the FA-TVP-VAR model.

Finally, we need to choose our prior beliefs about model change. The value of the forgetting factor α determines how fast model switches occur, and thus we use two values: $\alpha = 1$ which implies that we are implementing Bayesian model averaging (BMA) given data up to time t; and $\alpha = 0.99$ which implies that we implement dynamic model averaging (DMA) with relatively slowly varying model probabilities.

3.3 Estimates of the Financial Conditions Index

Before we proceed to the forecasting exercise, it is important to understand how both our estimation algorithm and model averaging work in the context of estimating an FCI. The results in this section are smoothed, that is estimated using the full sample of data from 1959Q1 - 2012Q1.

Figure 1 shows the factor estimates simply using all 20 financial variables without any model selection of model averaging being done. The estimates from the FAVAR, the FA-TVP-FAVAR and the TVP-FAVAR models are quite similar, especially during the first part of our sample. However. differences do exist, in particular specifically before, during, and after the recent financial crisis. Figure 1 also plots the principal component (PC) estimate of the 20 financial variables, and substantive differences are found between this and any of the FAVAR-based estimates. The FAVARs allow for time-varying covariances and VAR dynamics of the factor, while the principal component is a better approximation of factors coming from a homoskedastic static factor model. This characteristic explains why the FAVAR and PC estimates are on average similar, but differ more substantially at some peaks and troughs.

Figure 2 shows the impact of model averaging and selection on the estimate of the FCI, focussing on the TVP-FAVARs (but also including DMA done on the FAVARs for comparison). Although the broad patterns in the FCIs plotted in Figure 2 are similar, there are appreciable differences, particularly in the mid- to late 1980s and in the run-up to the financial crisis. DMA and BMA estimates based on the TVP-FAVARs tend to be quite similar to one another except for some periods early on in our sample. DMA estimates using the FAVARs are also quite similar to these, except for the run-up to the financial crisis. These are also, on average, similar to the FCI produced by the single TVP-FAVAR in Figure 1. However, after 1983, they differ quite substantially from the FCI produced by DMS. There is also a period of divergence of a lesser magnitude in the run-up to the financial crisis.

In Figure 3 we perform a comparison of the FCI constructed from dynamic model averaging of TVP-FAVAR models with the four existing FCIs maintained by four Federal Reserve Banks. First note that some of the indexes are actually measuring financial stress, or define tighter financial conditions using a positive value. For such FCI, we multiply by minus one, so that during the peak of the crisis all indexes are negative and, thus, comply with the shape of our FCIs. Additionally, to improve comparability, we standardize all the FCIs to have mean zero and variance one. If we standardize in this manner, it is interesting to note that it is our FCI (using DMA on the TVP-FAVARs) which achieves the minimum value in the depth of the financial crisis. In practice, what matters is the relative decrease of financial conditions during the recent financial crisis compared to normal periods, or other crises. In this regard, it is interesting to note that the Chicago Fed index (NFCI) predicts that the crises of the 1970s were deeper than the recent financial crisis. The Cleveland Fed index does not achieve a single minimum during the recent financial crisis, rather it has an inverted bell shape. These differences are quite substantial among all these FCIs, and (as we shall see) can have strong impact in forecasting.

Figures 1 through 3 compare a range of different FCI estimates. At this stage, we express no view on whether on whether any FCI is better or worse than any other. The key finding we stress is that, although they are similar to one another in many respects, substantive differences can occur. These differences are most notable when we compare our TVP-FAVAR or FAVAR-based estimates to conventional estimates (i.e. either PC of those produced by Federal Reserve Banks). Next in magnitude are the differences we find when comparing DMA and DMS approaches. Lowest in magnitude are the differences between TVP-FAVAR and FAVAR approaches indicating time variation in parameters is playing only a small role.

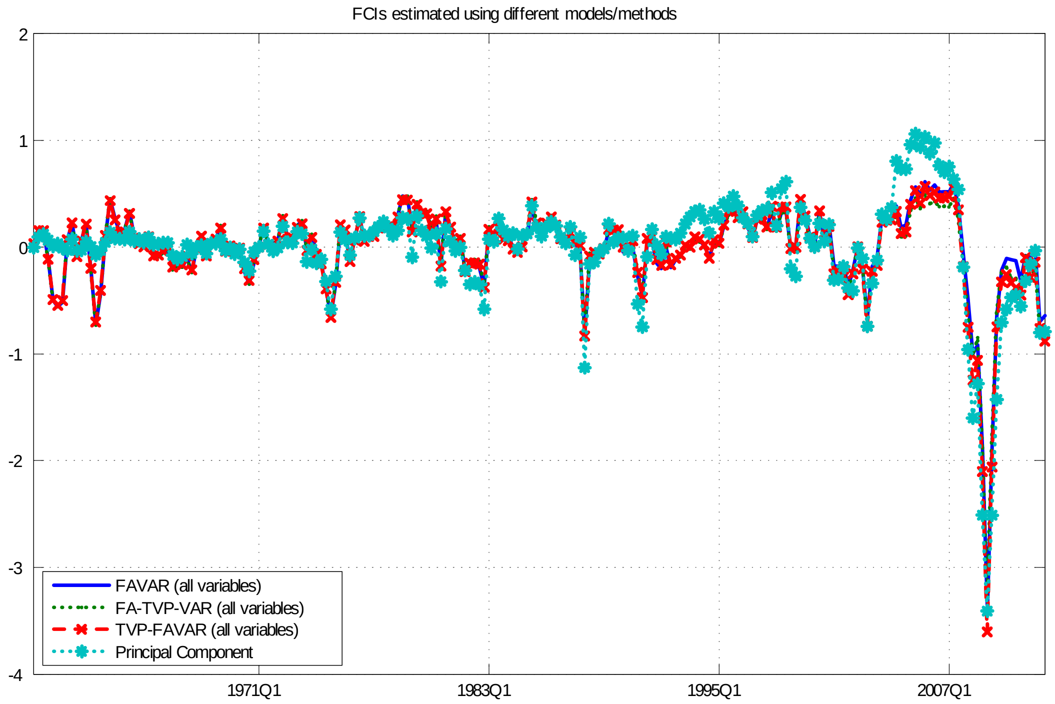


Figure 1. FCIs constructed from several versions of the heteroskedastic factor-augmented VAR model with all 20 financial variables used (no model averaging/selection in the loadings). For comparison, the principal component of the 20 financial variables is also plotted.

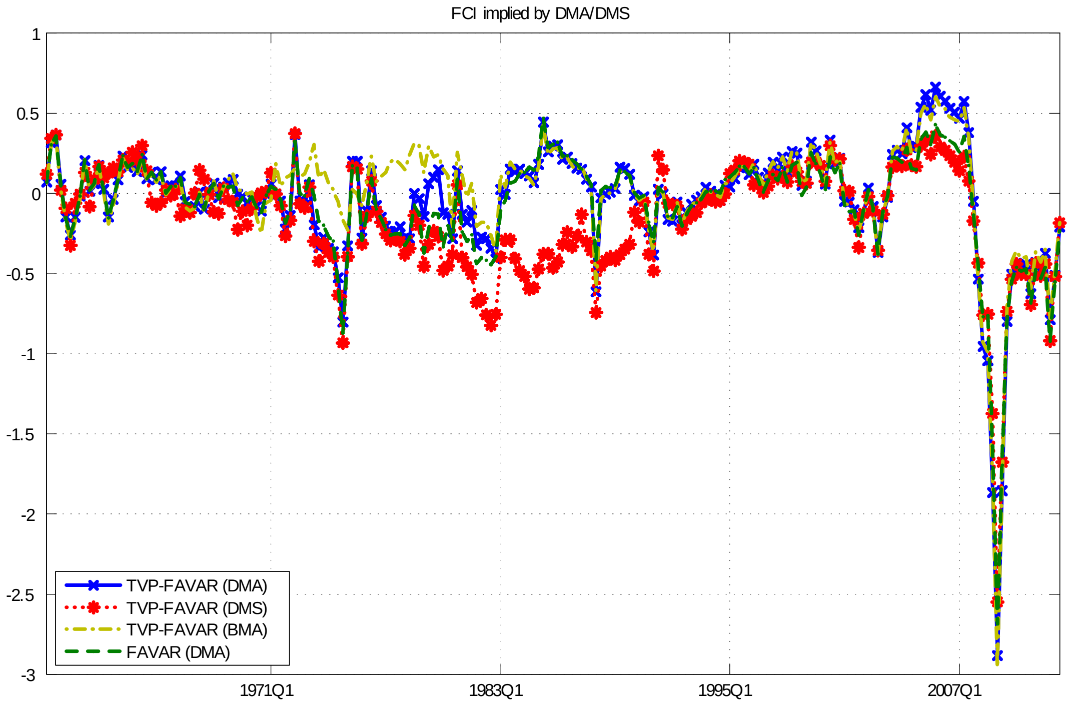


Figure 2. FCIs implied by BMA, DMA and DMS on the TVP-FAVARs (with DMA results for FAVARs provided for comparison)

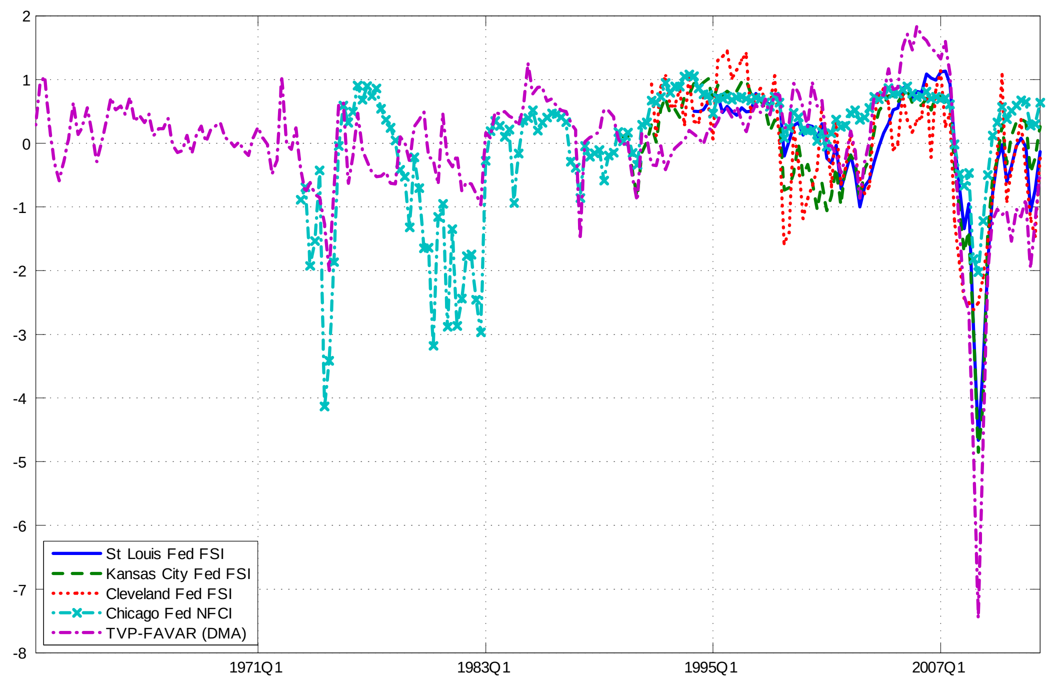


Figure 3. The FCI from the TVP-FAVAR with DMA compared to existing financial indexes maintained by four regional US central banks.

To provide some additional insight on what DMA is doing, we present Figures 4 and 5 which shed light on the number of variables selected when we do DMA or DMS on the TVP-FAVARs. In particular, Figure 4 calculates the expected number of variables used to extract the FCI at each point in time. If we denote by n_j the number of variables which load on the FCI under model M_j, then we calculate each time period the following expectation[8]

$$E\left(n_t^{DMA}\right) = \left(\sum_{j=1}^{J} \pi_{t|t,j} \times n_j\right) - 1.$$

Figure 4 shows that the number of variables used in DMA has increased over time until the late 90s, then dropped abruptly in early to mid 2000s, while it increased gradually until the peak of the recent financial crisis. DMA in the TVP-FAVAR implies that, in addition to the S&P 500, the FCI should include roughly 9 to 14 variables.

Figure 5 provides evidence on which variables receive most weight in the DMA procedure (or are selected by DMS). The numbers in each panel of this figure are the total probability DMA attaches to models which contain the variable named in the title on the panel. A pattern worth noting is that, consistent with Figure 4, many financial variables become important during and after the financial crisis. It is also worth noting that there is substantial variable switching. That is, there are a few variables which enter then abruptly leave (or vice versa) the FCI. In contrast to some of our previous work using regression models,[9] we find that DMA weights can change rapidly over time.

[8] We subtract one since the S&P500 variable is always included in all models.

[9] See Koop and Korobilis (2012), but also much of the Bayesian model averaging literature (e.g. the determinants of growth literature discussed in papers such as Eicher, Papageorgiou and Raftery, 2011).

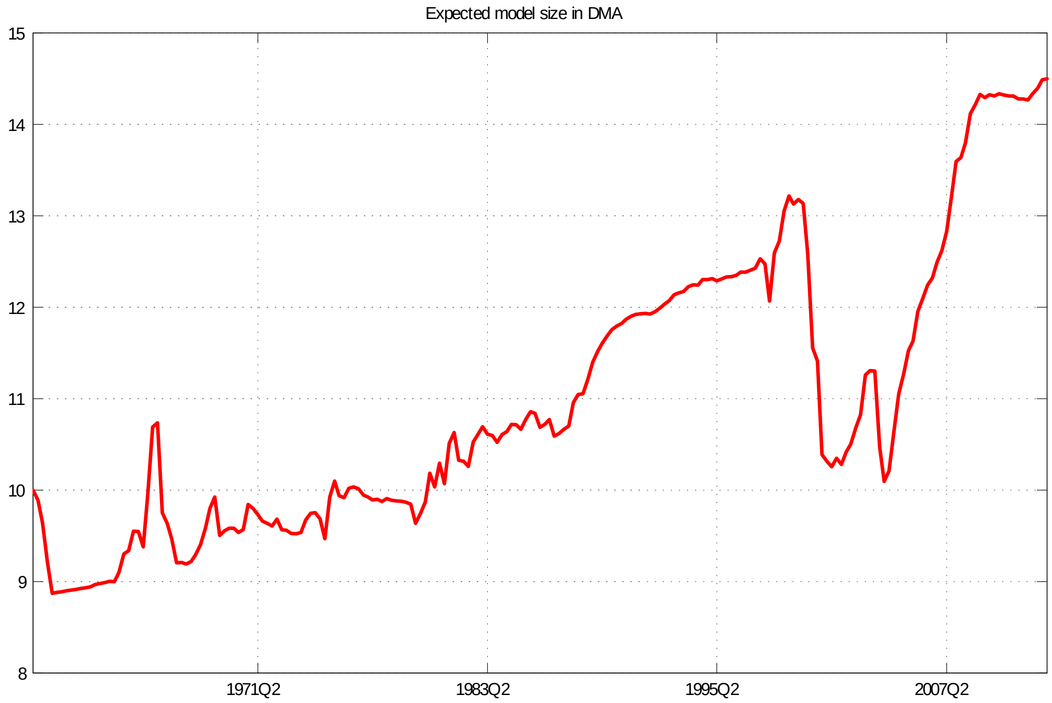
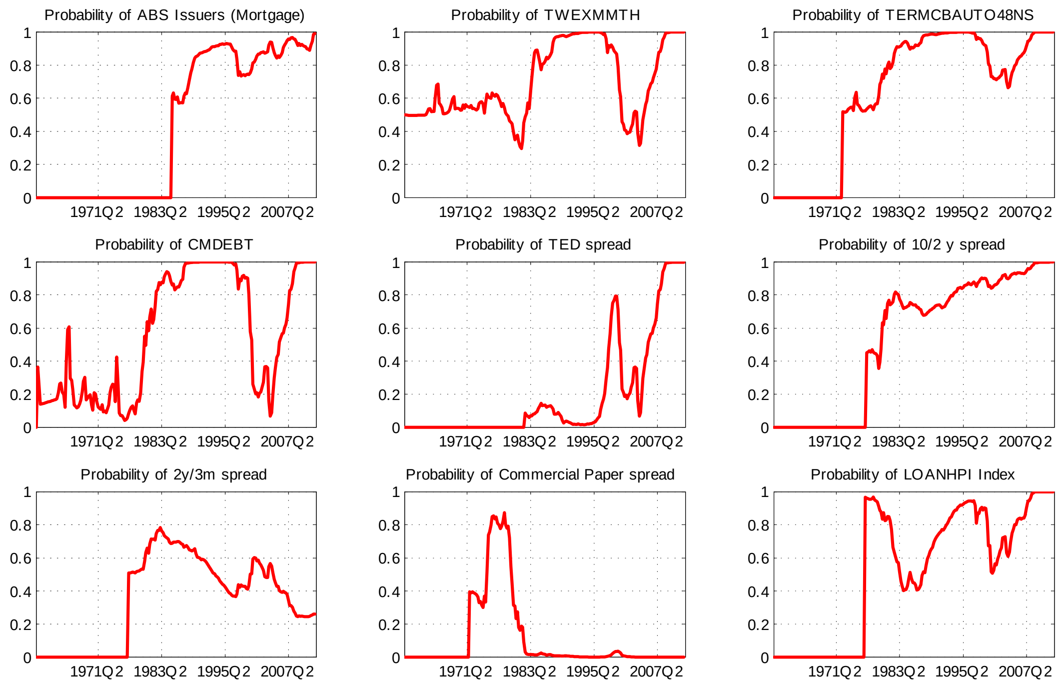


Figure 4. Average number of variables used to extract the FCI at each point in time as implied by DMA applied in the full TVP-FAVAR specification.



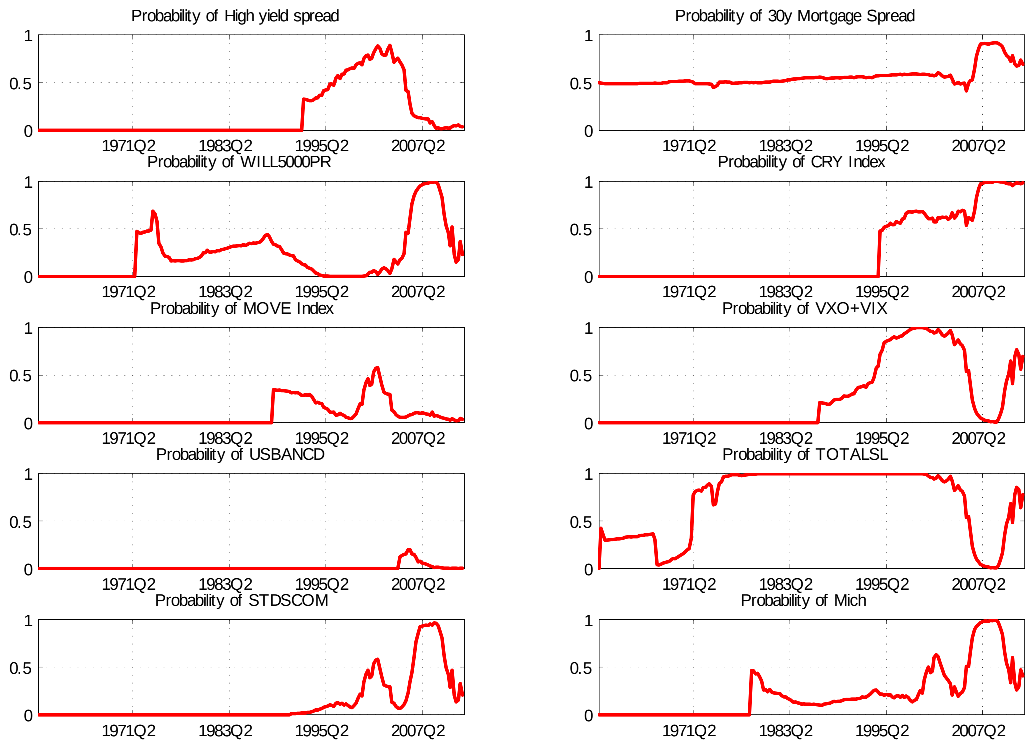


Figure 5. Time-varying probabilities of inclusion to the final FCI for each of the 19 financial variables (S&P500 is always included; see Section 3.1). Zero probabilities at the beginning of the sample for some of the variables correspond to periods of missing observations.

3.4 Forecasting

In this section, we investigate the performance of a wide range of models and methods for forecasting GDP growth and the unemployment rate. Our forecast evaluation period is 1990Q1 through 2012Q1-h for $h = 1, 2, 3, 4$ quarters ahead.[10] Evaluation of forecast accuracy is based on the mean squared forecast error (MSFE) divided by the MSFE produced by a TVP-VAR involving the five macroeconomic variables (not including any FCI).

Table 2 presents forecasting results for various FAVARs with or without time-variation in parameters and with and without DMA/DMS including: i) a VAR and TVP-VAR on the vector y_t of macroeconomic variables alone (no FCI added), ii) a VAR augmented with a principal component from all 20 financial variables, iii) FAVARs with all 20 financial variables included at all times (no DMA/DMS); iv) FAVARs with recursive BMA/BMS ($\alpha = 1$), and v) FAVARs with DMA/DMS ($\alpha = 0.99$). The main story is that our methods, which allow for time-variation in

[10]Forecasts for $h > 1$ are iterated.

parameters and the way model averaging or selection is done, forecast best. MSFEs are substantially lower than simple VAR or TVP-FAVARs or even the VAR augmented with a principal components estimate of the FCI.

We make the following observations:

- moving from the naive principal component to a specification which explicitly models factor dynamics and interaction with macro variables (such as our FAVAR with all 20 variables included), has large benefits for GDP growth rate forecasts; these benefits are less clear for unemployment rate forecasts.
- moving from the FAVAR to the FA-TVP-VAR or the full TVP-FAVAR, whether we also consider model averaging or not, has large benefits in forecasting. E.g. moving from the FAVAR (all variables) to the TVP-FAVAR (all variables) decreases the relative MSFE of GDP growth by 11%, while moving from the FAVAR (BMA) to the TVP-FAVAR (BMA) decreases the relative MSFE of unemployment by 9%.
- allowing for DMA/DMS or BMA/BMS also improves forecasting performance by up to 7-8% for GDP and up to 5% for unemployment, compared to the same model with all variables included. E.g. the difference of the relative MSFE of the FAVAR (all variables) with the FAVAR (BMA) is 4% for GDP and 3% for unemployment.
- selecting the best model, instead of averaging, seems to be the best strategy for GDP forecasts for $h = 1$ and 2 quarters ahead.

Table 2: Performance of our FCI based on various FAVAR models, 1990Q1 - 2012Q1

	GDP				UNEMPLOYMENT			
	h=1	h=2	h=3	h=4	h=1	h=2	h=3	h=4
VAR (no FCI)	1.27	1.29	1.10	1.02	2.21	2.25	2.06	1.90
TVP-VAR (no FCI)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
VAR + Principal Component	1.06	1.17	1.18	1.31	0.89	0.93	0.97	1.03
FAVAR (all variables)	0.92	1.03	1.07	1.07	0.93	0.96	1.00	1.03
FA-TVP-VAR (all variables)	0.81	0.90	0.95	0.99	0.88	0.83	0.83	0.85
TVP-FAVAR (all variables)	0.81	0.90	0.96	1.00	0.86	0.81	0.81	0.84
FAVAR (BMA)	0.88	0.98	1.02	1.03	0.90	0.94	0.98	1.02
FAVAR (BMS)	0.84	0.95	1.01	1.04	0.87	0.88	0.91	0.96
FA-TVP-VAR (BMA)	0.78	0.85	0.91	0.96	0.82	0.79	0.81	0.84
FA-TVP-VAR (BMS)	0.75	0.84	0.92	0.98	0.80	0.77	0.78	0.81
TVP-FAVAR (BMA)	0.77	0.85	0.91	0.96	0.81	0.78	0.79	0.82
TVP-FAVAR (BMS)	0.77	0.85	0.93	0.98	0.82	0.78	0.80	0.82
FAVAR (DMA)	0.88	0.98	1.02	1.03	0.90	0.93	0.98	1.01
FAVAR (DMS)	0.85	0.96	1.03	1.05	0.92	0.98	1.03	1.08
FA-TVP-VAR (DMA)	0.78	0.85	0.91	0.95	0.82	0.80	0.81	0.84
FA-TVP-VAR (DMS)	0.74	0.83	0.90	0.97	0.82	0.78	0.79	0.82
TVP-FAVAR (DMA)	0.77	0.85	0.91	0.96	0.81	0.78	0.80	0.83
TVP-FAVAR (DMS)	0.75	0.84	0.93	0.99	0.82	0.77	0.79	0.82

Note: FAVAR is the simple version of our model with all parameters constant. FA-TVP-VAR extends the FAVAR by adding time-variation in the VAR part (evolution of the factors). TVP-FAVAR is the full model where both VAR coefficients and loadings are time-varying. Dynamic Model Averaging (DMA) is implemented with forgetting factor $\alpha = 0.99$. Bayesian Model Averaging (BMA) is equivalent to DMA using $\alpha = 1$.

Judging the performance of point forecasts based on MSFE is only part of the picture regarding model performance. Predictive likelihoods can be used to evaluate the forecasting performance of the entire predictive distribution. In the present context, examination of predictive likelihoods is of particular interest since TVP models have many more parameters than their constant parameter variants, implying higher estimation error and, thus, higher forecast uncertainty. Furthermore, model averaging, whether done in a time-varying fashion or not, is expected to reduce uncertainty surrounding forecasts (see, e.g., Hoeting et al. 1999) relative to methods which use a single model.

Figure 6 contains several panels which plot the one-step ahead log-predictive likelihood (log-PL) of GDP growth for various models and methods for the forecast evaluation period 1990Q1-2012Q1. Note that TVP-FAVAR(DMA) is included in most panels to aid in visualizing the differences between the approaches. A major story of this figure is that doing DMA or DMS does lead to large improvements in predictive likelihoods, but this improvement happens mainly since the financial crisis.

It is also worth noting that time-variation in the parameters makes little difference in terms of predictive likelihoods when we are working with a single model including all 20 financial variables. However, time-variation in parameters does matter for DMA (e.g. TVP-FAVAR(DMA) almost always has higher predictive likelihoods than FAVAR(DMA)).

Remember that, with DMS we choose the model that has forecast best in the immediate past. While this strategy is optimal in normal times, when a rare event occurs this single best model might overfit past observations. The bottom left panel of the figure illustrates this. The log-PL of DMS is consistently above the log-PL of DMA (for the case of the TVP-FAVAR), but it is appreciably lower during the peak of the crisis. By averaging over many models, DMA can reduce the risk of this happening. Nevertheless, due to the time-varying nature of the DMA/DMS probabilities, the DMS algorithm adapts quickly after the deterioration in forecast performance of 2008Q4 and, after this point, its log-PL is again slightly higher than that of DMA. Finally, the bottom right panel of Figure 6 compares the log-PL of the full TVP-FAVAR model with and without DMA. In Table 2 we saw that DMA reduces MSFE, here we see if also leads to improved predictive likelihoods.

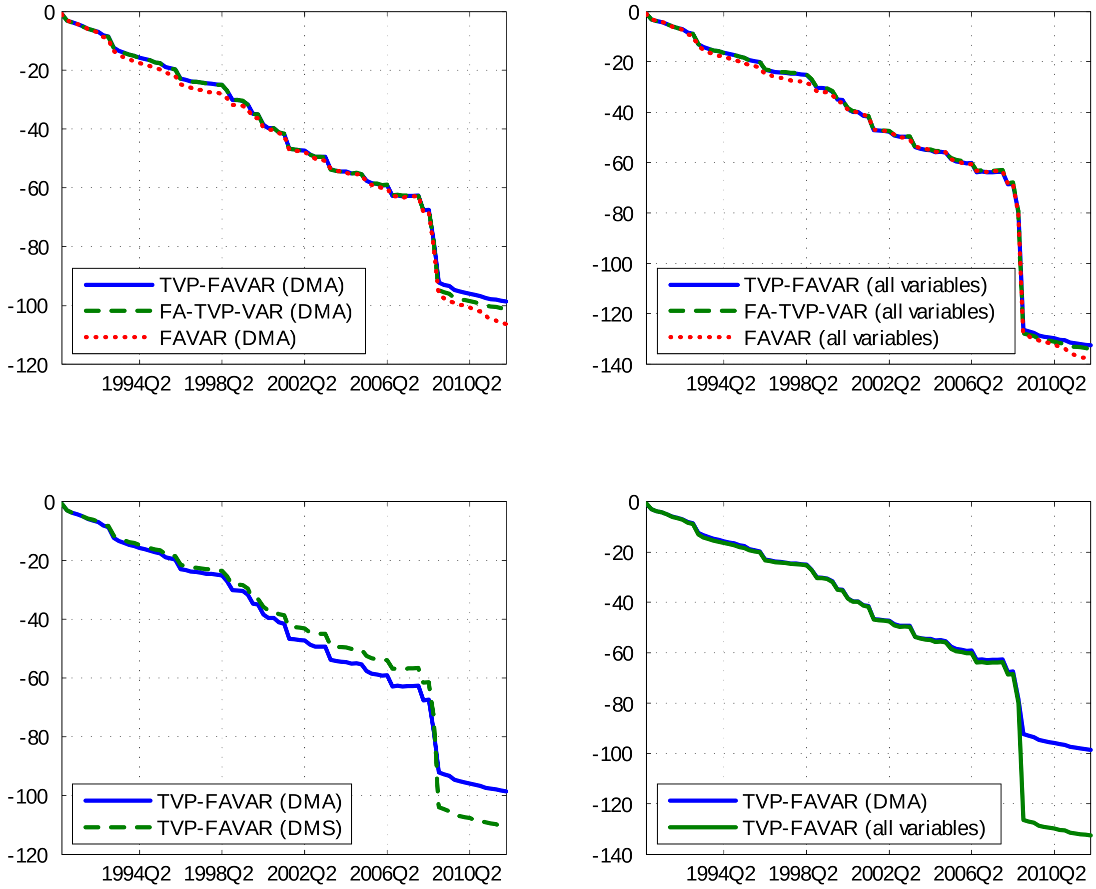


Figure 6. One step ahead ($h = 1$) cumulative sum of log-predictive likelihoods of GDP growth rate during the whole evaluation period 1990Q1-2012Q1, based on the various models we estimate.

Also of interest is the performance of our approach to VAR forecasts augmented with an existing FCI (see Table 1). Before doing so, we note that such comparisons are extremely difficult since different indexes are based on different assumptions, data transformations, frequencies and sample sizes. The earliest common starting date for the FCIs is 1994Q1 and, accordingly, we re-estimate our TVP-FAVAR (DMA) using data from this point and use 2000Q1 - 2012Q1-h as our forecast evaluation period.

Table 3 presents the MSFEs for FCI augmented VAR and TVP-VAR models, as well as the VAR and TVP-VAR with no FCI (just the five macro variables), for $h = 1, 2, 3, 4$ and for GDP and unemployment. All MSFEs are relative to the MSFE of the TVP-FAVAR (DMA) which is standardized to be one. Our index is doing very well in forecasting the unemployment rate, and is doing better than most indexes (the exception being the Chicago Fed FCI, in terms of 1-step ahead GDP forecasts). Note that the VAR and TVP-VAR models are used to construct the dynamics of our FCI (from the FAVARs) as well as construct forecasts of the macroeconomic variables, while the existing FCIs are constructed using different methods. This immediately gives an advantage to our FCI, however, this doesn't reduce the importance of the results presented in Table 3.

Table 3: Performance of our FCI compared to other FCIs, 2000Q1 - 2012Q1

	GDP				UNEMPLOYMENT			
	h=1	h=2	h=3	h=4	h=1	h=2	h=3	h=4
TVP-FAVAR (DMA)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
VAR (no FCI)	1.59	1.38	1.10	0.97	3.03	3.41	2.94	2.53
TVP-VAR (no FCI)	1.28	1.09	1.05	0.97	1.23	1.33	1.27	1.21
VAR (FCI 1)	1.22	1.15	1.11	0.91	1.66	2.36	2.25	1.99
TVP-VAR (FCI 1)	1.07	0.84	0.93	1.23	1.07	1.27	1.11	0.95
VAR (FCI 1)	1.04	1.24	1.16	1.01	1.95	2.24	2.21	2.18
TVP-VAR (FCI 2)	1.06	0.89	0.92	0.83	1.09	0.99	0.94	0.93
VAR (FCI 2)	1.28	1.38	1.19	0.97	1.52	1.59	1.44	1.27
TVP-VAR (FCI 3)	1.42	1.2	1.18	1.1	1.03	0.97	0.91	0.89
VAR (FCI 4)	0.92	1.05	1.05	0.97	1.13	1.22	1.19	1.12
TVP-VAR (FCI 4)	0.95	0.83	0.89	0.88	1.32	1.45	1.46	1.44

4 Conclusions

In this paper, we have argued for the desirability of constructing a dynamic financial conditions index which takes into account changes in the financial sector, its interaction with the macroeconomy and data availability. In particular, we want a methodology which can choose different financial variables at different points in time and weight them differently. We develop DMS and DMA methods, adapted from Raftery et al (2010) and others, to achieve this aim.

Working with a large model space involving many TVP-FAVARs (and restricted variants) which make different choices of financial variables, we find DMA and DMS methods lead to improve forecasts of macroeconomic variables, relative to methods which use a single model. This holds true regardless of whether the single model is parsimonious (e.g. a VAR for the macroeconomic variables) or parameter-rich (e.g. an unrestricted TVP-FAVAR which includes the same large set of financial variables at every point in time). The dynamic FCIs we construct are mostly similar to those constructed using conventional methods. However, particularly at times of great financial stress (e.g. the late 1970s and early 1980s and the recent financial crisis), our FCI can be quite different from conventional benchmarks. The DMA and DMS algorithm also indicates substantial inter-temporal variation in terms of which financial variables are used to construct it.

References

[1] Aiolfi, M., Capistrán, C., Timmermann, A., 2010. Forecast combinations. In: Michael P. Clements and David F. Hendry (Eds). The Oxford Handbook of Economic Forecasting. Oxford University Press, USA.

[2] Balakrishnan, R., Danninger, S., Elekdag, S., Tytell, I., 2009. The transmission of financial stress from advanced to emerging economies. IMF Working Papers 09/133, International Monetary Fund.

[3] Banerjee, A., Marcellino, M., Masten, I., 2008. Forecasting macroeconomic variables using diffusion indexes in short samples with structural change. CEPR Discussion Papers 6706, C.E.P.R. Discussion Papers.

[4] Bates, B. J., Plagborg-Møller, M., Stock, J. H., Watson, M. W., 2013. Consistent factor estimation in dynamic factor models with structural instability. Journal of Econometrics, Available online 17 April 2013, doi: 10.1016/j.jeconom.2013.04.014

[5] Beaton, K., Lalonde, R., Luu, C., 2009. A financial conditions index for the United States. Bank of Canada Discussion Paper, November.

[6] Bernanke, B., Boivin, J., Eliasz, P., 2005. Measuring monetary policy: A factor augmented vector autoregressive (FAVAR) approach. Quarterly Journal of Economics 120, 387-422.

[7] Boivin, J., Ng, S., 2006. Are more data always better for factor analysis? Journal of Econometrics 132, 169-194.

[8] Brave, S., Butters, R., 2011. Monitoring financial stability: a financial conditions index approach. Economic Perspectives, Issue Q1, Federal Reserve Bank of Chicago, 22-43.

[9] Breitung, J., Eickmeier, S., 2011. Testing for structural breaks in dynamic factor models. Journal of Econometrics, 163(1), 71-84.

[10] Clark, T., 2009. Real-time density forecasts from VARs with stochastic volatility. Federal Reserve Bank of Kansas City Research Working Paper 09-08.

[11] Cogley, T., Sargent, T. J., 2005. Drift and volatilities: Monetary policies and outcomes in the post WWII U.S. Review of Economic Dynamics 8(2), 262-302.

[12] Del Negro, M., Otrok, C., 2008. Dynamic factor models with time-varying parameters: Measuring changes in international business cycles. University of Missouri Manuscript.

[13] Doz, C., Giannone, D., Reichlin, L., 2011. A two-step estimator for large approximate dynamic factor models based on Kalman filtering. Journal of Econometrics 164, 188-205.

[14] Eicher, T. S., Papageorgiou, C., Raftery, A. E., 2011. Default priors and predictive performance in Bayesian model averaging, with application to growth determinants. Journal of Applied Econometrics 26, 30-55.

[15] Eickmeier, S., Lemke, W., Marcellino, M., 2011. The changing international transmission of financial shocks: evidence from a classical time-varying FAVAR. Deutsche Bundesbank, Discussion Paper Series 1: Economic Studies, No 05/2011.

[16] English, W., Tsatsaronis, K., Zoli, E., 2005. Assessing the predictive power of measures of financial conditions for macroeconomic variables. Bank for International Settlements Papers No. 22, 228-252.

[17] Giannone, D., Reichlin, L., Sala, L., 2005. Monetary Policy in Real Time. NBER Chapters, in: NBER Macroeconomics Annual 2004, Volume 19, pages 161-224, National Bureau of Economic Research, Inc.

[18] Gomez, E., Murcia, A., Zamudio, N., 2011. Financial conditions index: Early and leading indicator for Colombia? Financial Stability Report, Central Bank of Colombia.

[19] Hatzius, J., Hooper, P., Mishkin, F., Schoenholtz, K., Watson, M., 2010. Financial conditions indexes: A fresh look after the financial crisis. NBER Working Papers 16150, National Bureau of Economic Research, Inc.

[20] Hoeting, J. A., Madigan, D., Raftery, A. E., Volinsky, C. T., 1999. Bayesian Model Averaging: A Tutorial. Statistical Science 14, 382-417.

[21] Kárný, M., 2006. Optimized Bayesian Dynamic Advising: Theory and Algorithms, Springer-Verlag New York Inc.

[22] Kaufmann, S., Schumacher, C., 2012. Finding relevant variables in sparse Bayesian factor models: Economic applications and simulation results. Deutsche Bundesbank Discussion Paper No 29/2012.

[23] Koop, G., Korobilis, D., 2012. Forecasting inflation using dynamic model averaging. International Economic Review 53, 867-886.

[24] Koop, G., Korobilis, D., 2013. Large time-varying parameter VARs. Journal of Econometrics, Available online 17 April 2013, dx.doi.org/10.1016/j.jeconom.2013.04.007.

[25] Korobilis, D., 2013. Assessing the transmission of monetary policy shocks using time-varying parameter dynamic factor models. Oxford Bulletin of Economics and Statistics 75, 157-179.

[26] Kulhavý, R., Kraus, F., 1996. On duality of regularized exponential and linear forgetting. Automatica 32,1403-1416.

[27] Lütkepohl, H., 2005. New Introduction to Multiple Time Series Analysis. Springer: New York.

[28] Matheson, T., 2011. Financial conditions indexes for the United States and Euro Area. IMF Working Papers 11/93, International Monetary Fund.

[29] Nelson, L., Stear, E., 1976. The simultaneous on-line estimation of parameters and states in linear systems. IEEE Transactions on Automatic Control 21, 94 - 98.

[30] Primiceri. G., 2005. Time varying structural vector autoregressions and monetary policy. Review of Economic Studies 72, 821-852.

[31] Quintana, J.M., West, M., 1988. Time Series Analysis of Compositional Data. In Bayesian Statistics 3, (eds: J.M. Bernardo, M.H. De Groot, D.V. Lindley and A.F.M. Smith), Oxford University Press.

[32] Raftery, A. E., Karny, M., Ettler, P., 2010. Online prediction under model uncertainty via dynamic model averaging: Application to a cold rolling mill. Technometrics 52, 52-66.

[33] Schorfheide, F., Wolpin, K. E., 2012. On the Use of Holdout Samples for Model Selection. American Economic Review - Papers and Proceedings 102(3), 477-481.

[34] Stock, J. H., Watson, M. W., 2002. Macroeconomic forecasting using diffusion indexes. Journal of Business & Economic Statistics 20,147-162.

[35] Stock, J. H., Watson, M. W., 2009. Forecasting in dynamic factor models subject to structural instability. In: Jennifer Castle and Neil Shephard (Eds), The Methodology and Practice of Econometrics: A Festschrift in Honour of Professor David Hendry, Oxford: Oxford University Press.

A. Data Appendix

The following table describes the series we used to extract our Financial Conditions Index. The fourth column describes the stationarity transformation codes (Tcodes) which have been applied to each variable. Tcode shows the stationarity transformation for each variable: Tcode=1, variable remains untransformed (levels) and Tcode=5, use first log differences. The fifth column describes the source of each variable. The codes are: B - Bloomberg; D - Datastream; F - Federal Reserve Economic Data (http://research.stlouisfed.org/fred2/); G - Amit Goyal (http://www.hec.unil.ch/agoyal/); R - Board of Governors of the Federal Reserve System (http://www.federalreserve.gov/); U - University of Michigan (http://www.sca.isr.umich.edu/); W - Mark W. Watson (http://www.princeton.edu/ mwatson/).

Table A1: 20 variables used for forecasting

No	Mnemonic	Description	Tcode	Source	Sample
1	SP500	S&P 500 Stock Price Index	5	F	1959Q1 - 2012Q1
2	TWEXMMTH	FRB Nominal Major Currencies Dollar Index (Linked To EXRUS In 1973:1)	1	W	1959Q1 - 2012Q1
3	CMDEBT	Household Sector: Liabilities: Household Credit Market Debt Outstanding	5	F	1959Q1 - 2012Q1
4	30y Mortgage Spread	30y Conventional Mortgage Rate - 10y Treasury Rate	1	F	1959Q1 - 2012Q1
5	ABS Issuers (Mortgage)	Issuers Of Asset-Backed Securities; Total Mortgages	1	F	1984Q4 - 2012Q1
6	TERMCBAUTO48NS	Finance Rate On Consumer Installment Loans, New Autos 48 Month Loan	1	F	1972Q1 - 2012Q1
7	TED spread	3m LIBOR - 3m Treasury Bill Rate	1	F	1981Q4 - 2012Q1
8	10/2 y spread	10-Year/2-Year Treasury Yield Spread	1	F	1976Q3 - 2012Q1
9	2y/3m spread	2-Year/3-Month Treasury Yield Spread	1	F	1976Q3 - 2012Q1
10	Commercial Paper spread	3-Month Financial Commercial Paper/Treasury Bill Spread	1	B	1971Q2 - 2012Q1
11	LOANHPI Index	Home Loan Performance Index U.S. Index Level	5	B	1976Q1 - 2012Q1
12	High yield spread	BofA Merrill Lynch US High Yield Master II Effective Yield - Moody's BAA	1	F	1992Q1 - 2012Q1
13	WILL5000PR	Wilshire 5000 Price Index	5	F	1971Q1 - 2012Q1
14	CRY Index	Thomson Reuters/Jefferies CRB Commodity Index	1	B	1994Q1 - 2012Q1
15	MOVE Index	Merrill Lynch One-Month Treasury Options Volatility Index (MOVE)	1	B	1988Q2 - 2012Q1
16	VXO+VIX	CBOE (S&P100 + S&P500) Volatility Index	1	B	1986Q3 - 2012Q1
17	USBANCD	US Banks Sector CDS Index 5Y - CDS Prem. Mid	1	D	2004Q1 - 2012Q1
18	TOTALSL	Total Consumer Credit Owned And Securitized, Outstanding	5	F	1959Q1 - 2012Q1
19	STDSCOM	SLOOS: Percentage of banks tightening standards for real estate loans	1	R	1990Q3 - 2012Q1
20	Mich	Michigan Survey: Expected Change In Financial Situation	1	U	1978Q1 - 2012Q1

B. Technical Appendix

In this appendix, we describe the econometric methods we use to estimate a TVP-FAVAR (and restricted versions of it).

We write the TVP-FAVAR compactly as

$$\begin{aligned} x_t &= z_t \Lambda_t + u_t, \quad u_t \sim N(0, V_t) & \text{(B.1)} \\ z_t &= z_{t-1} \beta_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, Q_t) & \text{(B.2)} \\ \lambda_t &= \lambda_{t-1} + v_t, \quad v_t \sim N(0, W_t) & \text{(B.3)} \\ \beta_t &= \beta_{t-1} + \eta_t, \quad \eta_t \sim N(0, R_t) & \text{(B.4)} \end{aligned}$$

where $\lambda_t = \left((\lambda_t^y)', \left(\lambda_t^f\right)' \right)'$ and $z_t = \begin{bmatrix} y_t \\ f_t \end{bmatrix}$. We also use notation where $\widetilde{f}_t$ is the standard principal components estimate of f_t based on x_t (using data up to time t) and $\widetilde{z}_t = \begin{bmatrix} y_t \\ \widetilde{f}_t \end{bmatrix}$. If a_t is a vector then $a_{i,t}$ is the i^{th} element of that vector; and if A_t is a matrix $A_{ii,t}$ is its $(i,i)^{th}$ element.

Our estimation algorithm requires initialization of all state variables. In particular we define the following initial conditions for all system unknown parameters

$$\begin{aligned} f_0 &\sim N\left(0, \Sigma_{0|0}^f\right), & \text{(B.5)} \\ \lambda_0 &\sim N\left(0, \Sigma_{0|0}^\lambda\right), & \text{(B.6)} \\ \beta_0 &\sim N\left(0, \Sigma_{0|0}^\beta\right), & \text{(B.7)} \\ V_0 &\equiv 1 \times I_n, & \text{(B.8)} \\ Q_0 &\equiv 1 \times I_{s+1}. & \text{(B.9)} \end{aligned}$$

The algorithm follows the following steps:

1. Given the initial conditions and $z_t = \widetilde{z}_t$ obtain **filtered** estimates of $\lambda_t, \beta_t, V_t, Q_t$ using the following recursion for $t = 1, .., T$

   (a) The Kalman filter tells us:

   $$\begin{aligned} \lambda_t | Data_{1:t-1} &\sim N\left(\lambda_{t|t-1}, \Sigma_{t|t-1}^\lambda\right), \\ \beta_t | Data_{1:t-1} &\sim N\left(\beta_{t|t-1}, \Sigma_{t|t-1}^\beta\right), \end{aligned}$$

   where $\lambda_{t|t-1} = \lambda_{t-1|t-1}$, $\Sigma_{t|t-1}^\lambda = \Sigma_{t-1|t-1}^\lambda + \widehat{W}_t$, $\beta_{t|t-1} = \beta_{t-1|t-1}$ and $\Sigma_{t|t-1}^\beta = \Sigma_{t-1|t-1}^\beta + \widehat{R}_t$. The error covariances are estimated using forgetting factors as: $\widehat{W}_t = \left(1 - \kappa_3^{-1}\right) \Sigma_{t-1|t-1}^\lambda$ and $\widehat{R}_t = \left(1 - \kappa_4^{-1}\right) \Sigma_{t-1|t-1}^\beta$.

(b) Calculate estimates of V_t and Q_t for use in the updating step using the following EWMA specifications:

$$\widehat{V}_{i,t} = \kappa_1 V_{i,t-1|t-1} + (1-\kappa_1)\,\widehat{u}_{i,t}\widehat{u}'_{i,t} \tag{B.10}$$

$$\widehat{Q}_t = \kappa_2 Q_{t-1|t-1} + (1-\kappa_2)\,\widehat{\varepsilon}_t\widehat{\varepsilon}'_t \tag{B.11}$$

where $\widehat{u}_{i,t} = x_{i,t} - \widetilde{z}_t\lambda_{i,t|t-1}$, for $i = 1, ..., n$, and $\widehat{\varepsilon}_t = \widetilde{z}_t - \widetilde{z}_{t-1}\beta_{t|t-1}$.

(c) Update λ_t and β_t given information at time t using the Kalman filter update step

- Update $\lambda_{i,t}$ for each $i = 1, ..., n$ using

$$\lambda_{it}|Data_{1:t} \sim N\left(\lambda_{i,t|t}, \Sigma^{\lambda}_{ii,t|t}\right),$$

where $\lambda_{i,t|t} = \lambda_{i,t|t-1} + \Sigma^{\lambda}_{ii,t|t-1}\widetilde{z}'_t\left(\widehat{V}_{ii,t} + \widetilde{z}_t\Sigma^{\lambda}_{ii,t|t-1}\widetilde{z}'_t\right)^{-1}\left(x_t - \widetilde{z}_t\lambda_{t|t-1}\right)$ and $\Sigma^{\lambda}_{ii,t|t} = \Sigma^{\lambda}_{ii,t|t-1} - \Sigma^{\lambda}_{ii,t|t-1}\widetilde{z}'_t\left(\widehat{V}_{ii,t} + \widetilde{z}_t\Sigma^{\lambda}_{ii,t|t-1}\widetilde{z}'_t\right)^{-1}\widetilde{z}_t\Sigma^{\lambda}_{ii,t|t-1}$.

- Update β_t from

$$\beta_t|Data_{1:t} \sim N\left(\beta_{t|t}, \Sigma^{\beta}_{t|t}\right),$$

where $\beta_{t|t} = \beta_{t|t-1} + \Sigma^{\beta}_{t|t-1}\widetilde{z}'_{t-1}\left(\widehat{Q}_t + \widetilde{z}_{t-1}\Sigma^{\beta}_{t|t-1}\widetilde{z}'_{t-1}\right)^{-1}\left(\widetilde{z}_t - \widetilde{z}_{t-1}\widehat{\beta}_{t|t-1}\right)$ and $\Sigma^{\beta}_{t|t} = \Sigma^{\beta}_{t|t-1} - \Sigma^{\beta}_{t|t-1}\widetilde{z}'_{t-1}\left(\widehat{Q}_t + \widetilde{z}_{t-1}\Sigma^{\beta}_{t|t-1}\widetilde{z}'_{t-1}\right)^{-1}\widetilde{z}_{t-1}\Sigma^{\beta}_{t|t-1}$.

(d) Update V_t and Q_t given information at time t using the EWMA specifications as follows:

$$V_{i,t|t} = \kappa_1 V_{i,t-1|t-1} + (1-\kappa_1)\,\widehat{u}_{i,t|t}\widehat{u}'_{i,t|t} \tag{B.12}$$

$$Q_{t|t} = \kappa_2 Q_{t-1|t-1} + (1-\kappa_2)\,\widehat{\varepsilon}_{t|t}\widehat{\varepsilon}'_{t|t} \tag{B.13}$$

where $\widehat{u}_{i,t|t} = x_{i,t} - \widetilde{z}_t\lambda_{i,t|t}$, for $i = 1, ..., n$, and $\widehat{\varepsilon}_{t|t} = \widetilde{z}_t - \widetilde{z}_{t-1}\beta_{t|t}$.

2. Obtain **smoothed** estimates of $\lambda_t, \beta_t, V_t, Q_t$ using the following recursions for $t = T-1, .., 1$

(a) Update λ_t and β_t given information at time $t+1$ using the fixed interval smoother

- Update $\lambda_{i,t}$ for each $i = 1, ..., n$ from

$$\lambda_{it}|Data_{1:T} \sim N\left(\lambda_{i,t|t+1}, \Sigma^{\lambda}_{ii,t|t+1}\right),$$

where $\lambda_{i,t|t+1} = \lambda_{i,t|t} + C^{\lambda}_t\left(\lambda_{i,t+1|t+1} - \lambda_{i,t+1|t}\right)$, $\Sigma^{\lambda}_{ii,t|t+1} = \Sigma^{\lambda}_{ii,t|t} + C^{\lambda}_t\left(\Sigma^{\lambda}_{ii,t+1|t+1} - \Sigma^{\lambda}_{ii,t+1|t}\right)C^{\lambda\prime}_t$ and $C^{\lambda}_t = \Sigma^{\lambda}_{ii,t|t}\left(\Sigma^{\lambda}_{ii,t+1|t}\right)^{-1}$.

- Update β_t from

$$\beta_t | Data_{1:T} \sim N\left(\beta_{t|t+1}, \Sigma^{\beta}_{t|t+1}\right),$$

where $\beta_{t|t+1} = \beta_{t|t} + C^{\beta}_t \left(\beta_{t+1|t+1} - \beta_{t+1|t}\right)$, $\Sigma^{\beta}_{t|t+1} = \Sigma^{\beta}_{t|t} + C^{\beta}_t \left(\Sigma^{\beta}_{t+1|t+1} - \Sigma^{\beta}_{t+1|t}\right) C^{\beta\prime}_t$ and $C^{\beta}_t = \Sigma^{\beta}_{t|t}\left(\Sigma^{\beta}_{t+1|t}\right)^{-1}$.

(b) Update V_t and Q_t given information at time $t+1$ using the following equations

$$V^{-1}_{t|t+1} = \kappa_1 V^{-1}_{t|t} + (1-\kappa_1) V^{-1}_{t+1|t+1}, \tag{B.14}$$

$$Q^{-1}_{t|t+1} = \kappa_2 Q^{-1}_{t|t} + (1-\kappa_2) Q^{-1}_{t+1|t+1}. \tag{B.15}$$

3. Means and variances of f_t given appropriate estimates of $\lambda_t, \beta_t, V_t, Q_t$ described in the preceding steps can be obtained using the standard **Kalman filter and smoother.**

Treatment of missing values

In our application our sample is unbalanced, since it contains many financial variables which have been collected only after the 1970s or the 1980s. Similar issues are faced by organizations which monitor FCIs. For instance, the Chicago Fed National FCI comprises 100 series where most of them have different starting dates. Although specific computational methods for dealing with such issues exist (e.g. the EM algorithm, or Gibbs sampler with data augmentation), our focus is on averaging over many models which means such methods are computationally infeasible. Accordingly, similar to our purpose of developing a simulation-free and fast algorithm for parameter estimation, we want to avoid simulation methods for estimating the missing data in x_t. Additionally, methods such as interpolation can work poorly when missing values are at the beginning of the sample.

Since the missing data in x_t are in the beginning, we make the assumption that the factor (FCI) is estimated using only the observed series. The estimation algorithm above allows for such an approach in a straightforward manner by just replacing missing values with zeros. The loadings λ (whether time-varying, or constant) will become equal to 0, thus removing from the estimate of f_t the effect of the variables in x_t which have missing values at time t. This feature holds both for the initial principal components estimate $\widetilde{f}_t$, as well as the final Kalman filter estimate.

Estimation of a single TVP-FAVAR (and its variants)

Given the algorithm above, we can estimate the TVP-FAVAR by choosing values of $\kappa_1, \kappa_2, \kappa_3, \kappa_4 < 1$. The main results in the paper set these to be equal to 0.99. The restricted special cases of the TVP-FAVAR listed in Section 2.1 can be obtained by setting forgetting and/or decay factors to particular values. If we set $\kappa_3 = 1$ and $\kappa_4 < 1$ then we obtain the FA-TVP-VAR. Setting $\kappa_3 = 1$ and $\kappa_4 = 1$ leads to the FAVAR. Note that if we also set $\kappa_1 = \kappa_2 = 1$ we can estimate homoskedastic versions of the various models, since in that case $V_t = V_{t-1} = ... = V_1 = V_0$ and $Q_t = Q_{t-1} = ... = Q_1 = Q_0$. Nevertheless, as we comment in the main body of the paper, this is a case which is always dominated (in terms of forecast performance) by the heteroskedastic case.

Estimation of multiple models (DMA/DMS)

In order to implement the DMA/DMS exercises we run the algorithm described above for each of the $2^{19} = 524,288$ models. Note that for a specific DMA exercise all models are nested, and the only thing that changes is the number of variables in the vector x_t that we use in order to extract the FCI. Given our discussion about how missing values are treated by the Kalman filter, in order to estimate a specific model which uses, say, the 1st, 3rd and 15th series in x_t, we simply multiply all but the 1st, 3rd and 15th columns of x_t with zeros. In that case, we remove at all times t the effects of all 17 variables we do not use for estimation of the specific model, and at the same time we still have as a dependent variable a 20×1 dimensional vector (and programming is greatly simplified).

The most important feature of DMA is that, unlike many Bayesian model selection and averaging procedures based on Markov Chain Monte Carlo methods, there is no dependence in estimating each model and iterations using "for" loops are independent. That means that it is trivial to adapt our code to use features such as parallel computing, thus taking advantage of the widespread availability of modern multi-core processors (or large clusters of PCs). In MATLAB this is as easy as replacing the typical "for" loop which would run for models 1 to $524,288$, with a "parfor" loop.

The reader is encouraged to look at our code which is available on https://sites.google.com/site/dimitriskorobilis/matlab, which also has the option to call the Parallel Processing Toolbox in a MATLAB environment.

C. Sensitivity analysis

In this section we quote further results for several different choices of prior hyperparameters, which can reflect different beliefs about time variation in the model parameters as well as beliefs about variation over time in the optimal model. Our benchmark prior specification was based on non-informative choices which are always quite appealing. Such noninformative choices are easily implementable since our estimation algorithm does not rely on simulation (MCMC) and it is numerically very stable.

C.1. Comparison of a relatively noninformative with training sample priors

In the body of the paper, results are presented for a subjectively-elicited but relatively noninformative prior. An interesting alternative to such a prior is to choose prior hyperparameters using a training sample of data. In the context of TVP-VARs, Primiceri (2005) suggests such a prior which is based on splitting the data into a training sample (or holdout sample) and a testing set (or estimation sample). OLS or other simple estimates of all model parameters in the training sample are used as starting values for the testing sample. Such training sample priors are commonly-used in Bayesian analysis, and in the context of time-varying parameter models help provide regularized posterior estimators which can also help numerical stability. This latter feature is important in the case of TVP-VARs and TVP-FAVARs estimated with MCMC - see the discussion in Section 4.1 of Primiceri (2005).

In this section we introduce such a training sample prior for our FAVAR, FA-TVP-VAR and TVP-FAVAR models. We use the first 20 years of data (1959Q1-1978Q4) in our original sample as the training sample. We estimate a FAVAR with constant parameters using OLS methods (and replacing the factor with the principal component estimate) on this training sample. These OLS estimates are used to in the initial conditions for estimation sample, 1979Q1-2012Q1, as follows

$$
\begin{aligned}
f_0 &\sim N\left(\widehat{f}_T^{TS}, 0.1\right), \\
\lambda_0 &\sim N\left(0, 4 \times var\left(\widehat{\lambda}^{TS}\right)\right), \\
\beta_0 &\sim N\left(0, 4 \times var\left(\widehat{\beta}^{TS}\right)\right), \\
V_0 &\equiv 1 \times \widehat{V}^{TS}, \\
Q_0 &\equiv 1 \times \widehat{Q}^{TS}, \\
\pi_{0|0,j} &= \frac{1}{J},
\end{aligned}
$$

where parameters with a hat and a superscript TS denote OLS estimates of the respective parameters in the time-invariant FAVAR fitted using the training sample.

Other settings used in the main body of the paper remain the same, e.g. we use four lags everywhere and the decay and forgetting factors that define each of the models (DMA vs BMA, or FAVAR vs FA-TVP-VAR vs TVP-FAVAR) are exactly the ones specified in Section 3.2. The forecast evaluation sample is 1990Q1-2012Q1, and we use the one-step ahead log predictive likelihood to evaluate forecasting performance.

Table C1: Average predictive likelihoods, $h = 1$, 1990Q1-2012Q1

	Subjective priors		Training sample priors	
	GDP	unemployment	GDP	unemployment
FAVAR BMA	-1.491	-8.231	-2.555	-13.375
FAVAR BMS	-1.576	-8.801	-2.673	-13.747
FA-TVP-FAVAR BMA	-1.439	-8.079	-2.693	-13.408
FA-TVP-FAVAR BMS	-1.480	-8.216	-2.868	-13.654
TVP-FAVAR BMA	-1.424	-7.789	-2.697	-13.393
TVP-FAVAR BMS	-1.494	-7.918	-2.856	-13.655
FAVAR DMA	-1.490	-8.225	-2.538	-13.364
FAVAR DMS	-1.498	-8.805	-2.691	-13.747
FA-TVP-FAVAR DMA	-1.439	-8.074	-2.660	-13.387
FA-TVP-FAVAR DMS	-1.473	-8.216	-2.880	-13.654
TVP-FAVAR DMA	-1.424	-7.783	-2.665	-13.369
TVP-FAVAR DMS	-1.488	-7.918	-2.862	-13.655

In Table C1 we see that forecast performance, as measured by the mean of the one-step ahead log-predictive likelihood over the period 1990Q1-2012Q1, is inferior for all models estimated with a training sample prior. This result suggests our subjective prior is a sensible one, consistent with the pattern in the data. That is, combining the subjective prior with the complete sample of data means we have more information, leading to good forecast performance. By using a training sample in order to inform initial conditions, we lose observations from the likelihood function and do not include the subjective prior information. As Schorfheide and Wolpin (2012) note:

> "[...] from a Bayesian perspective the use of holdout samples is suboptimal because the computation of posterior probabilities should be based on the entire sample and not just on a subsample." Schorfheide and Wolpin (2012)

From our own experience with Bayesian VARs (and FAVARs), we can argue that training sample priors are very important in cases where numerical stability is an issue. For example, the decision of Primiceri (2005) and others to use training sample priors when estimating TVP-VAR with MCMC works well because serious numerical issues can occur when noninformative priors and diffuse initial conditions are used in the full sample. In the present paper where we examine high dimensional TVP-FAVARs, we do not have numerical issues due to the computational simplicity of our algorithm (which does not involve use of MCMC methods). This advantage of our estimation methods justifies our decision to use relatively noninformative priors in the full available sample 1959-2012 as the benchmark case.

C.2. Comparison of faster/slower model switching

Calculation of DMA/DMS time-varying probabilities depends on selection of a hyperparameter α, which is a forgetting factor that determines how fast we "forget" past observations. Thus, this hyperparameter α controls how much data we use for the calculation of time-varying probabilities at time t and, thus, determines the rate of model switching. That is, the less data that is used, the more rapidly will model switching will occur.

Remember that $\alpha = 1$ leads to standard BMA, where all data available up to time t are used to calculate the model probabilities $\pi_{t|t,j}$, $j = 1, ..., J$. The benchmark results reported in the body of the paper, $\alpha = 0.99$, allows for slightly more rapid model switching. In this appendix we provide results for a choice of α which reflects beliefs about even faster model switches ($\alpha = 0.95$), as well as the extreme case of $\alpha = 0.001$. Note that this latter case is almost equivalent to the case of model averaging using equal weights, since it is trivial to prove that as $\alpha \rightarrow 0$ then $\pi_{t|t-1,j} \rightarrow 1/J$ for all $j = 1, ..., J$. Naive model averaging schemes using equal weights have been shown in many cases to perform much better than more elaborate econometric techniques that perform estimation of the model averaging weights; see Aiolfi,Capistrán and Timmermann (2010).

Table C2 shows the log-predictive likelihoods for the full TVP-FAVAR model for the different values of α. For all values of α DMA is performs better than DMS. The case $\alpha = 0.95$ is always dominated by our benchmark value of $\alpha = 0.99$. Interestingly, the equal-weights case ($\alpha = 0.001$) leads to more precise forecasts for GDP (but not unemployment). Therefore, there could potentially be further improvements in forecast accuracy by estimating α using grid-search methods. We remind the reader that we do not perform such a search due to the already high computational demands of our empirical exercise.

Table C2: Average predictive likelihoods, $h = 1$, 1990Q1-2012Q1

	GDP	unemployment
BENCHMARK SPECIFICATION (WITH $\alpha = 0.99$)		
TVP-FAVAR DMA	-1.424	-7.783
TVP-FAVAR DMS	-1.494	-7.918
MODELS WITH FASTER CHANGING PROBABILITIES		
TVP-FAVAR DMA ($\alpha = 0.95$)	-1.487	-7.847
TVP-FAVAR DMS ($\alpha = 0.95$)	-1.707	-8.032
TVP-FAVAR DMA ($\alpha = 0.001$)	-1.414	-7.848
TVP-FAVAR DMS ($\alpha = 0.001$)	-1.621	-7.977

C.3. Comparison of faster/slower parameter switching

Similar to the hyperparameters that control model switching, $\kappa_1, \kappa_2, \kappa_3, \kappa_4$ control the amount of time-variation in the error covariances (V_t, Q_t), as well as the time-varying loadings λ_t and the VAR coefficients β_t. Table C3 presents results for different choices of these decay and forgetting factors.

Table C3: Average predictive likelihoods, $h = 1$, 1990Q1-2012Q1

	GDP	unemployment
BENCHMARK SPECIFICATION		
TVP-FAVAR DMA	-1.424	-7.783
TVP-FAVAR DMS	-1.494	-7.918
MODELS WITH FASTER CHANGING PARAMETERS		
Case 1: faster moving volatility		
TVP-FAVAR DMA ($\kappa_1 = \kappa_2 = 0.92$)	-1.514	-7.362
TVP-FAVAR DMS ($\kappa_1 = \kappa_2 = 0.92$)	-1.684	-7.519
Case 2: faster moving coefficients		
TVP-FAVAR DMA ($\kappa_3 = \kappa_4 = 0.97$)	-1.527	-7.260
TVP-FAVAR DMS ($\kappa_3 = \kappa_4 = 0.97$)	-1.717	-7.434
Case 3: faster moving volatility & coefficients		
TVP-FAVAR DMA ($\kappa_1 = \kappa_2 = 0.92, \kappa_3 = \kappa_4 = 0.97$)	-1.689	-7.304
TVP-FAVAR DMS ($\kappa_1 = \kappa_2 = 0.92, \kappa_3 = \kappa_4 = 0.97$)	-1.796	-7.518